SOLUTION

[2] 1. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning. Let $a, b, c \in \mathbb{Z}$.

	True	False	
(a)	○	⌀	For all $x \in \mathbb{R}$, $\lfloor x \rfloor < \lceil x \rceil$.
(b)	○	⌀	If $a \mid (b+c)$, then $a \mid b$ or $a \mid c$.
(c)	⌀	○	If $b \in \mathbb{Z}$ is an odd prime, then $b+5$ is composite.
(d)	⌀	○	If a and b are relatively prime, then $\mathrm{lcm}(a, b) = ab$.

[2] 2. Find the base 8 representation of 2025.

repeated division:

$2025 = 253 \cdot 8 + 1$

$253 = 31 \cdot 8 + 5$

$31 = 3 \cdot 8 + 7$

$3 = 0 \cdot 8 + 3$

so $2025 = (3751)_8$

OR

greedy algorithm

$2025 = 3 \cdot 512 + 489$

$= 3 \cdot 8^3 + 7 \cdot 64 + 41$

$= 3 \cdot 8^3 + 7 \cdot 8^2 + 5 \cdot 8 + 1$

so $2025 = (3751)_8$

[3] 3. Let $a, b \in \mathbb{Z}$. Prove that if $10 \mid a$ and $14 \mid b$, then $35 \mid ab$.

Suppose $10 \mid a$ and $14 \mid b$, then $\exists\, k, l \in \mathbb{Z}$ such that $a = 10k$ and $b = 14l$. Then

$$ab = (10k)(14l) = 140kl = 35(4kl)$$

Since $4kl \in \mathbb{Z}$, we have $35 \mid ab$.

[2] 4. Let $n = 2^{60} \cdot 5^{42}$. Using the Fundamental Theorem of Arithmetic, explain why there exists no integer k such that $n = 18k$.

FTA says that the prime factorization of a positive integer is unique. Since $3 \mid 18$ then if $n = 18k$ we will have $3 \mid n$, but $n = 2^{60} \cdot 5^{42}$ and 3 does not appear in the prime factorization of n. Therefore, $n \neq 18k$ for any $k \in \mathbb{Z}$.

[4] 5. Use the Euclidean algorithm to compute $\gcd(629, 272)$. Then use the Euclidean algorithm backwards to find $x, y \in \mathbb{Z}$ such that $\gcd(629, 272) = 629x + 272y$.

$629 = 2 \cdot 272 + 85$

$272 = 3 \cdot 85 + 17$

$85 = 5 \cdot 17 + 0$

so $\gcd(629, 272) = 17$

$17 = 272 - 3 \cdot 85$

$= 272 - 3(629 - 2 \cdot 272)$

$= 272 - 3 \cdot 629 + 6 \cdot 272$

$= 7 \cdot 272 - 3 \cdot 629$

so $x = -3$, $y = 7$

[2] 6. Suppose $n \equiv -13 \pmod 9$. What is the remainder when $n^2 + 82n$ is divided by 9?

$n \equiv -13 \equiv 5 \pmod 9$

$n^2 + 82n \equiv n^2 + n$ since $82 \equiv 1 \pmod n$

$\equiv 5^2 + 5$

$\equiv 30$

$\equiv 3 \pmod 9 \Rightarrow$ remainder is 3