SOLUTION

[2] 1. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning. Let $\mathcal{R}$ be a relation on the set $A = \{1, 2, 3\}$.

	True	**False**	
(a)	○	⌀	Let B and C be sets. If $B \times C = C \times B$, then $B = C$.
(b)	⌀	○	If $(3, 3) \notin \mathcal{R}$, then $\mathcal{R}$ is not reflexive.
(c)	○	⌀	If $(1, 2) \in \mathcal{R}$ and $(2, 1) \in \mathcal{R}$, then $\mathcal{R}$ is symmetric.
(d)	⌀	○	It is possible for a relation to be both symmetric and antisymmetric.

[3] 2. Consider the relation $\mathcal{R}$ on $\mathbb{Q}$ defined by $(a, b) \in \mathcal{R}$ if and only if $2(a - b)$ is an integer. Show that $\mathcal{R}$ is transitive.

Suppose $(a,b) \in \mathcal{R}$ and $(b,c) \in \mathcal{R}$, so $2(a-b) = k$ and $2(b-c) = l$ for some $k, l \in \mathbb{Z}$. Then

$$\begin{aligned} 2(a-c) &= 2(a-b) + 2(b-c) \\ &= k + l \in \mathbb{Z} \end{aligned}$$

so $2(a-c)$ is an integer. Therefore $(a,c) \in \mathcal{R}$, and $\mathcal{R}$ is transitive.

[2] 3. Define a relation $\mathcal{R}$ on the positive integers by $(a, b) \in \mathcal{R}$ if and only if ab is a perfect square (so ab is the square of an integer). The relation $\mathcal{R}$ is an equivalence relation (you do not need to show this). List four elements that are in the equivalence class of 3, i.e. list four elements in [3].

$$\begin{aligned} [3] &= \{x : (3,x) \in \mathcal{R}\} \\ &= \{x : 3x \text{ is a perfect square}\} \\ &= \{3n^2 : n \in \mathbb{Z}\} \end{aligned}$$

This is the full equivalence class, you just needed to list 4 things in the set

[3] 4. Let $A = \mathbb{Z} \setminus \{0\}$, the set of non-zero integers. Consider the function $f : A \to \mathbb{Q}$ defined by $f(x) = \frac{x+1}{x}$. Prove that f is injective (one-to-one).

Let $x, y \in A$. Suppose $f(x) = f(y)$. Then

$$\frac{x+1}{x} = \frac{y+1}{y} \Rightarrow y(x+1) = x(y+1)$$

$$xy + y = xy + x$$

$$x = y$$

so f is injective.

[3] 5. Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$, and $C = \{4, 5, 6\}$. Let $f : A \to B$ be defined by $f = \{(1, c), (2, d), (3, a)\}$. Find a function $g : B \to C$ so that $g \circ f$ is surjective (onto). Describe each of the functions g and $g \circ f$ as sets of ordered pairs.

Lots of options – here is one:

$$g = \{(a, 4), (b, 4), (c, 5), (d, 6)\}$$

$$g \circ f = \{(3, 4), (1, 5), (2, 6)\}$$

need
- a, c, d to map to distinct elts in C by g
- b to map somewhere by g

[2] 6. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning.

	True	**False**	
(a)	⊘	○	If a function f is surjective, then its codomain and range are equal.
(b)	⊘	○	If f has an inverse, then f is a bijection.
(c)	⊘	○	The function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 3x + 2$ is injective.
(d)	○	⊘	In order for the composition $f \circ g$ to be defined, we must have range$(f) \subseteq$ domain(g).