MATH 122 (A02) **Quiz 5**

Date: Apr. 3, 2025 **Time Limit: 25 minutes**

This quiz contains 4 pages (including this one, and a scrap page) and 6 questions.

Before starting your quiz, enter your name and student ID (clearly) on this page.

You may not open the quiz until instructed to do so. In the meantime, please read the following instructions:

- This quiz is closed-book. You are permitted a Sharp EL-510R series calculator (but it probably won't be very useful).
- Please organize your work reasonably neatly in the space provided. Work that is scattered and difficult to follow, without clear ordering, will receive little credit.
- Unless otherwise specified, in order to receive full credit for a correct solution you must show your work. Unsupported answers will not receive full credit, even if they are correct.

Question:	1	2	3	4	5	6	Total
Points:	2	3	2	3	3	2	15

[2] 1. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning. Let $\mathcal{R}$ be a relation on the set $A = \{1, 2, 3\}$.

	True	**False**	
(a)	◯	◯	Let B and C be sets. If $B \times C = C \times B$, then $B = C$.
(b)	◯	◯	If $(3, 3) \notin \mathcal{R}$, then $\mathcal{R}$ is not reflexive.
(c)	◯	◯	If $(1, 2) \in \mathcal{R}$ and $(2, 1) \in \mathcal{R}$, then $\mathcal{R}$ is symmetric.
(d)	◯	◯	It is possible for a relation to be both symmetric and antisymmetric.

[3] 2. Consider the relation $\mathcal{R}$ on $\mathbb{Q}$ defined by $(a, b) \in \mathcal{R}$ if and only if $2(a - b)$ is an integer. Show that $\mathcal{R}$ is transitive.

[2] 3. Define a relation $\mathcal{R}$ on the positive integers by $(a, b) \in \mathcal{R}$ if and only if ab is a perfect square (so ab is the square of an integer). The relation $\mathcal{R}$ is an equivalence relation (you do not need to show this). List four elements that are in the equivalence class of 3, i.e. list four elements in [3].

[3] 4. Let $A = \mathbb{Z} \setminus \{0\}$, the set of non-zero integers. Consider the function $f : A \to \mathbb{Q}$ defined by $f(x) = \frac{x+1}{x}$. Prove that f is injective (one-to-one).

[3] 5. Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$, and $C = \{4, 5, 6\}$. Let $f : A \to B$ be defined by $f = \{(1, c), (2, d), (3, a)\}$. Find a function $g : B \to C$ so that $g \circ f$ is surjective (onto). Describe each of the functions g and $g \circ f$ as sets of ordered pairs.

[2] 6. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning.

	True	**False**	
(a)	○	○	If a function f is surjective, then its codomain and range are equal.
(b)	○	○	If f has an inverse, then f is a bijection.
(c)	○	○	The function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 3x + 2$ is injective.
(d)	○	○	In order for the composition $f \circ g$ to be defined, we must have range$(f) \subseteq$ domain(g).

This page is for rough work only, and will not be graded.
If you have work on this page you want marked,
please indicate this next to the original question.