201501 Math 122 [~~A02~~ A03] Midterm #1

#V00:_______________ Name: Solutions

This midterm has 4 pages and 11 questions. There are 30 marks available. The time limit is 50 minutes. Calculators will not help with these questions. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [3] Find all combinations of truth values for p, q and r for which the statement $\neg p \leftrightarrow (q \wedge (q \to \neg r))$ is true.

p false $\Rightarrow$ $\neg p$ true $\therefore$ $q \wedge (q \to \neg r)$ true
$\therefore$ q true & r false.

p true $\Rightarrow$ $\neg p$ false $\therefore$ $q \wedge (q \to \neg r)$ false
$\therefore$ q false & r anything
OR q true & r true

$$\therefore \begin{array}{c} p \quad q \quad r \\ \left(\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}\right) \end{array}$$

2. Suppose the universe is the integers. Consider the following (correct) argument:

   Suppose n is a multiple of 5. Then $n = 5k$ for some integer k. Hence, $n^2 = (5k)^2 = 25k^2 = 5(5k^2)$. Therefore, n^2 is a multiple of 5.

   (a) [1] Write the implication proved by the given argument in plain English.

   If n is a multiple of 5, then n^2 is a multiple of 5.

   (b) [2] Write the contrapositive of your statement in (a). Is it also proved by the given argument?

   If n^2 is not a multiple of 5, then n is not a multiple of 5.
   An implication is L.E. to its contrapositive $\therefore$ Yes

3. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   T The contrapositive of $p \to \neg q$ is $q \to \neg p$.

   T If q is true, then so is $p \to q$.

   F The converse of $\forall x, (x^2 \leq 0) \to (x = 0)$ is $\exists x, (x \neq 0) \to (x^2 > 0)$.

   T The negation of "*Some golf shots are hooks or slices*" is "*Every golf shot is neither a hook nor a slice*".

4. Suppose the universe is $\{3, 5\}$. For each of the following, write a logically equivalent statement that does not contain any quantifiers.

   (a) [1] $\forall x, (x^2 > 16)$

   $(3^2 > 16) \wedge (5^2 > 16)$

   (b) [2] $\forall x, \exists y, x + y = 2x$

   $[(3+3 = 2\cdot 3) \vee (3+5 = 2\cdot 3)]$
   $\wedge\ [(5+3 = 2\cdot 5) \vee (5+5 = 2\cdot 5)]$

5. [3] Find an expression logically equivalent to $(p \to \neg q) \vee r$ which uses only the symbols $p, q, r, \wedge, \neg$ and brackets.

$(p \to \neg q) \vee r \Leftrightarrow (\neg p \vee \neg q) \vee r$ Known LE.

$\Leftrightarrow \neg(p \wedge q) \vee r$ DeMorgan

$\Leftrightarrow \neg[(p \wedge q) \wedge \neg r]$ ''

6. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

T The statement $\neg p \to p$ is neither a tautology nor a contradiction.

T If two statements are logically equivalent, then so are their negations.

T If $p \Rightarrow q$ and $\neg p \Rightarrow \neg q$ then p and q are logically equivalent.

T If the premises of an argument can not all be true, then the argument is valid.

7. [4] Use known logical equivalences to show that $(p \vee q) \wedge (\neg p \vee \neg q)$ is logically equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$.

$(p \vee q) \wedge (\neg p \vee \neg q)$

$\Leftrightarrow [(p \vee q) \wedge \neg p] \vee [(p \vee q) \wedge \neg q]$ Dist've.

$\Leftrightarrow [\cancel{(p \wedge \neg p)} \vee (q \wedge \neg p)]$

0

$\vee [(p \wedge \neg q) \vee \cancel{(q \wedge \neg q)}]$ ″

0

$\Leftrightarrow (q \wedge \neg p) \vee (p \wedge \neg q)$ Known taut. & identity

$\Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$ Comm. x2

8. [3] Use known logical equivalences and inference rules to show that the following argument is valid.

$$\begin{array}{l} b \\ \neg a \leftrightarrow b \\ \neg c \rightarrow a \\ \hline \therefore\ c \end{array}$$

1. b Premise
2. $\neg a \leftrightarrow b$ ″
3. $\neg c \rightarrow a$ ″
4. $(\neg a \rightarrow b) \wedge (b \rightarrow \neg a)$ 3, LE.
5. $b \rightarrow \neg a$ 4, Conj've Simp.
6. $\neg a \rightarrow c$ 3, Contrapos.
7. $b \rightarrow c$ 5, 6, Chain Rule
8. $\therefore c$ 1, 7, M.P.

9. [3] Write the argument below in symbolic form, and then give a counterexample to show that it is not valid. Remember to define the letters you use to represent statements.

$$\begin{array}{l} \text{If I watch football, then I don't do mathematics} \\ \text{If I do mathematics, then I watch hockey} \\ \hline \therefore \text{ If I don't watch hockey, then I watch football} \end{array}$$

Let:

f: I watch football
m: I do mathematics
h: I watch hockey.

The argument is:

$$\begin{array}{l} f \rightarrow \neg m \\ m \rightarrow h \\ \hline \therefore \neg h \rightarrow f \end{array}$$

For the truth assignment

$$\begin{pmatrix} f & h & m \\ 0 & 0 & 0 \end{pmatrix}$$

all premises are true and the conclusion false.
$\therefore$ The argument is not valid.

10. [2] Let A, B and C be sets such that $A \subseteq B$ and $B \subseteq C$. Prove that $A \subseteq C$ using an argument that starts with "*Take any* $x \in A$.".

Take any $x \in A$.
Since $A \subseteq B$, $x \in B$.
Since $B \subseteq C$, $x \in C$.
$\therefore$ $A \subseteq C$.

11. [2] Let $A = \{1, \{2, 3\}, 3\}$ and $B = \{1, 2, \{3\}\}$. Use the blank to indicate whether each statement is true or false. No reasons are necessary.

F $A = B$.
F $2 \in A$.
T $\emptyset \subsetneq A$.
T $B \in \mathcal{P}(B)$.