201601 Math 122 [A01] Midterm #1

#V00:________________ Name: Key

This midterm has 4 pages and 10 questions. There are 30 marks available. The time limit is 50 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [3] Find all combinations of truth values for p, q and r for which the statement $\neg p \leftrightarrow (q \wedge \neg(p \to r))$ is true.

p true ⇒ want q false or ¬(p→r) false
∴ q false or r true

p false ⇒ want q true and ¬(p→r) true
∴ want q false and p→r false
Not possible b/c p is false.

∴ The relevant combinations are
$\begin{pmatrix} p & q & r \\ 1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} p & q & r \\ 1 & 0 & 1 \end{pmatrix}$ & $\begin{pmatrix} p & q & r \\ 1 & 1 & 1 \end{pmatrix}$

2. Suppose the universe is the integers. Consider the following (correct) argument:

   Suppose n and k are odd. Then $n = 2t + 1$ for some integer t, and $k = 2\ell + 1$ for some integer ℓ. Hence, $nk = (2t + 1)(2\ell + 1) = 4t\ell + 2t + 2\ell + 1$. Therefore, nk is odd.

   (a) [1] Write the implication proved by the given argument in plain English.

   If n and k are odd, then nk is odd

   (b) [2] Write the converse of your statement in (a). Is it also proved by the argument?

   If nk is odd, then n is odd and k is odd
   No. It is not logically equivalent to the statement proved.

3. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   _T_ The contrapositive of $p \to \neg q$ is $q \to \neg p$.

   _F_ The converse of *"If Origi scores, then Liverpool win"* is *"If Liverpool lose or tie, then Origi does not score"*.

   _F_ The negation of *"If I had a million dollars, I'd buy you a fur coat"* is *"If I don't have a million dollars, I won't buy you a fur coat"*.

   _T_ If q is false, then $p \to \neg q$ is true.

4. [3] Suppose the universe is the integers. Determine the truth value of each statement, and briefly justify your answer.

   (a) $\forall m, [(m \neq 0) \wedge (m^2 < 1)] \rightarrow (2m = 3)$. TRUE

   The hypothesis of the implication is always false, so the implication is always true

   (b) $\exists m, \forall n, mn = 0$. TRUE

   If $m=0$, then $mn=0$ for all n.

5. [3] Find an expression logically equivalent to $\neg(p \rightarrow \neg q) \vee \neg r$ which uses only the symbols $p, q, r, \wedge, \neg$ and brackets.

$\neg(p \rightarrow \neg q) \vee \neg r$

$\Leftrightarrow \neg(\neg p \vee \neg q) \vee \neg r$ Known LE.

$\Leftrightarrow (p \wedge q) \vee \neg r$ DeMorgan &, Dbl Neg'n x2

$\Leftrightarrow \neg(\neg(p \wedge q) \wedge r)$ DeMorgan

6. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

T The statement $p \wedge (p \rightarrow \neg p)$ is a contradiction.

F The negation of "*Every prime number is odd*" is "*No even number is prime*".

T The statement "*If the integer n is at least 2, then it has a prime divisor*" contains a hidden universal quantifier, and a hidden existential quantifier.

F If s_1 logically implies s_2, and $\neg s_2$ logically implies $\neg s_1$, then s_1 and s_2 are logically equivalent.

7. [4] Use known logical equivalences to show that $(p \vee q) \wedge (\neg p \vee \neg q)$ is logically equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$.

$(p \vee q) \wedge (\neg p \vee \neg q)$

$\Leftrightarrow ((p \vee q) \wedge \neg p) \vee ((p \wedge q) \vee \neg q)$ Dist

$\Leftrightarrow [(p \wedge \neg p) \vee (q \wedge \neg p)]$

$\vee [(p \wedge \neg q) \vee (q \wedge \neg q)]$ Dist

$\Leftrightarrow [0 \vee (q \wedge \neg p)] \vee [(p \wedge \neg q) \vee 0]$ Known Contradictions

$\Leftrightarrow (q \wedge \neg p) \vee (p \wedge \neg q)$ Identity x 2

$\Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$ Comm. x 2

8. [3] Use known logical equivalences and inference rules to show that the following argument is valid.

$$\begin{array}{rl} & b \\ & \neg a \leftrightarrow b \\ & \neg c \rightarrow a \\ \hline \therefore & c \end{array}$$

1. b Premise
2. $\neg a \leftrightarrow b$ Premise
3. $(\neg a \rightarrow b) \wedge (b \rightarrow \neg a)$ 2, L.E.
4. $b \rightarrow \neg a$ 3, Conj. Simpl.
5. $\neg a$ 1, 4, M.P.
6. $\neg c \rightarrow a$ Premise
7. $\neg a \rightarrow c$ 6, Contrapositive
8. c 5, 7, M.P.

9. [3] Write the argument below in symbolic form, and then give a counterexample to show that it is not valid. Remember to define the letters you use to represent statements.

I exercise, or rest on the couch
If I am not watching golf and not watching football, then I did not exercise
I am not resting on the couch
I am not watching football

∴ I am not watching golf

Let e be "I exercise"
r be "I rest on the couch"
g be "I watch golf"
f be "I watch football"

The argument is

$$\begin{array}{l} e \vee r \\ (\neg g \wedge \neg f) \to \neg e \\ \neg r \\ \neg f \\ \hline \therefore \neg g \end{array}$$

The truth assignment $\begin{pmatrix} e & r & g & f \\ 1 & 0 & 1 & 0 \end{pmatrix}$ makes all premises true & the conclusion false ∴ the argument is not valid.

10. A sign posted at an amusement park says "*In order to ride the roller coaster, you must be at least 1.3 metres tall, or at least 18 years old. If you are not at least 1.3 metres tall, or not at least 18 years old, then you can not ride the roller coaster.*"

(a) [2] Write the two statements on the sign in symbolic form. Make sure to specify which symbols represent which statements.

Let r be "you can ride the roller coaster"
t be "you are ≥ 1.3 m tall"
e be "you are ≥ 18 years old"

Then stmt 1 is $r \to (t \vee e)$
& " 2 is $(\neg t \vee \neg e) \to \neg r$

(b) [2] Give an example of a situation where the two statements on the sign have different truth values. Which statement places more restrictions on who can ride the roller coaster? Explain.

- Suppose you are 17 years old and 1.5 m tall, and you can ride the roller coaster.
- stmt 1 is true
  stmt 2 is false
- stmt 2 imposes more restrictions; the contrapositive of 1 is $(\neg t \wedge \neg e) \to \neg r$.
- Since $(\neg t \wedge \neg e) \Rightarrow (\neg t \vee \neg e)$, but these stmts are not L.E., 2 is more restrictive