202109 Term Test 2
Math 122 A05
Instructor: Natasha Morrison

First name (please write as legibly as possible within the boxes)

Last name

V number (without V)

Please write your V number without the "V"
Do not open the booklet before you are told to

Date and Time: Friday, November 26, 2021 at 13.30pm.

Instructions: There are 10 pages and 7 questions. There are 24 marks available. The time limit is 45 minutes. Math and Stats standard calculators are allowed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit.
Please count your pages and report any discrepancy immediately to the invigilator.

True/False Instructions: Question 1 consists of 12 true/false questions labelled **TF 1** to **TF 12**. The last page of your test booklet is a bubble sheet for answering them. You can detach the back page from the rest of the test. Only fill in a bubble for questions 1-12 on the bubble sheet. When making your selection, **True is A** and **False is B**. Do not select C, D or E. Do not select more than one bubble.

Do not use the back of the bubble sheet for rough work.

Nothing written on this page will be graded.

1. [6] Use the **bubble sheet** provided on the last page of the test booklet to indicate whether each statement is **True (A)** or **False (B)**.

   [1] $1 + 5^1 + 5^2 + \cdots + 5^n = \frac{1}{4}(5^{n+1} - 1)$.

   [2] When using induction to prove a statement $S(n)$ for all $n \geq 0$, in the induction hypothesis you assume $S(n)$ is true for all n.

   [3] Let $a_0, a_1, \ldots$ be the sequence recursively defined by $a_0 = 1$, $a_1 = 3$ and $a_n = 2a_{n-1} - a_{n-2}$ for all $n \geq 2$. Then $a_4 = 7$.

   [4] If a statement $S(n)$ is true for every $n \in \{1, 2, \ldots, k\}$, then $S(n)$ is true when $n = k + 1$.

   [5] Let $a, b \in \mathbb{Z}$. If $\gcd(a, b) = 1$, then there are no integers k, ℓ such that $ka + \ell b = 1$.

   [6] $36^{35} - 343^{123} \equiv 1$ (modulo 7)

   [7] $(221)_3 = 17$.

   [8] There are integers a and b such that $2 \cdot 3^a = 6 \cdot 5^b$.

   [9] Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. Then $(b, 2) \in A \times B$.

   [10] $A \times B = B \times A$ only when $A = B$.

   [11] Let $A = \{1, 2, 3, 4\}$. The relation on A defined by $\mathcal{R} = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ is reflexive.

   [12] Let $\mathcal{R}$ be the relation on $\mathbb{N}$ defined by $a \sim b$ in $\mathcal{R}$ if and only if $\frac{a}{b} \leq \frac{b}{a}$. $\mathcal{R}$ is antisymmetric.

2. [5] Use induction to prove that $3 + 6 + 9 + \cdots + 3n = \frac{3n(n+1)}{2}$, for all $n \geq 1$.

3. [3] Let $a, b, d \in \mathbb{Z} \setminus \{0\}$. Prove that if $d|b$, then $ad|ab^2$.

4. (a) [2] Use the Euclidean algorithm to find $\gcd(1234, 567)$.

(b) [1] Use your solution from part (a) to find $\text{lcm}(1234, 567)$.

5. [4] Let $\mathcal{R}$ be the relation on $\mathbb{Z}$ defined by $a \sim b$ if and only if $3|a-b$. Prove that $\mathcal{R}$ is an equivalence relation.

6. [1] Let $a, d \in \mathbb{R} \setminus \{0\}$. Write a recursive definition for the sequence $a, a+d, a+2d, a+3d, \ldots$.

7. [2] Let A, B, C be sets. Prove that $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$.

Name: _____ **Student ID Number:** _____

Instructions:

Please completely fill in the rectangle associated with your response. Example: ■ ☐B☐ ☐C☐ ☐D☐ ☐E☐

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