202401 Math 122 A01 Quiz #1

#V__________________ Name: ____Key____

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a calculator even though doing so is unnecessary and unhelpful. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F).

   __F__ There are truth values for p and q so that the statements $p \wedge q$, $p \wedge \neg q$ and $\neg p \wedge q$ are all true.

   __T__ If $s_1 \Rightarrow s_2$ and $\neg s_1 \Rightarrow \neg s_2$, then $s_1 \Leftrightarrow s_2$.

   __F__ The converse of $\neg p \to q$ is $\neg q \to p$.

   __F__ An argument is valid if there is a truth assignment so that the premises and conclusion are all true.

2. For given integers m and n, let the statements p, q, r be defined by:

$$p : m \text{ is even}; \quad q : n \text{ is even}; \quad r : m+n \text{ is even}.$$

   (a) [1] Write the statement "if m is even and n is odd, then $m+n$ is odd" in symbolic form. (Note: these are specific integers so quantifiers are not needed.)

   $(p \wedge \neg q) \to r$

   (b) [1] Write the negation of the statement "if m is odd and n is odd, then $m+n$ is odd" in plain English.

   m is odd and n is odd, but m+n is even

   (c) [1] Write the contrapositive of the statement $(r \wedge p) \to \neg q$ in plain English.

   if n is even then m+n is odd or m is odd

3. [2] Find an expression logically equivalent to $\neg(p \to \neg q) \vee \neg r$ which uses only the symbols $p, q, r, \wedge, \neg$ and brackets. Give reasons for each logical equivalence used.

$\neg(p \to \neg q) \vee \neg r$

$\Leftrightarrow \neg(\neg p \vee \neg q) \vee \neg r$ Known LE

$\Leftrightarrow (p \wedge q) \vee \neg r$ DeMorgan

$\Leftrightarrow \neg(\neg(p \wedge q) \wedge r)$ DeMorgan

4. [3] Use known logical equivalences and inference rules to show the argument below is valid. Give reasons for each step.

$$\begin{array}{l} \neg c \vee s \\ \neg c \to w \\ \neg w \\ \hline \therefore \; s \end{array}$$

1.	$\neg c \vee s$	Premise
2.	$\neg c \to w$	"
3.	$\neg w$	"
4.	$c \to s$	1, LE
5.	$\neg w \to c$	2, Contrapositive
6.	$\neg w \to s$	4,5 chain rule
7.	s	6, 3 M.P.

5. Suppose the universe is the integers.

(a) [1] Write the statement $\forall x, \exists y, y > x$ in plain English.

There is no largest integer

(b) [1] Write the statement $\exists y, \forall x, y \geq x$ in plain English.

There is a largest integer

(c) [1] One of the statements in parts (a) and (b) is true and the other is false. Which one is false? Briefly explain why it is false.

(b) is false. For any integer y, the integer $y+1 > y$. Thus no integer can be largest.

6. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F). No reasons are necessary.

T If x can be any real number, then the statement $\exists x, (x^2 \leq 0) \to (x = -1)$ is true.

F The negation of "*All students like calculus*" is "*Some students like calculus*".

F For the universe of integers, when the statement "*if m is even, then $m \cdot n$ is even*" is written in symbols, then only universal quantifiers appear.

T The negation of $\forall x, \exists y, x^2 < y^2$ is $\exists x, \forall y, x^2 \geq y^2$.