

202401 Math 122 A01 Quiz #1

#V _____

Name: _____

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a calculator even though doing so is unnecessary and unhelpful. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F).

- _____ There are truth values for p and q so that the statements $p \wedge q$, $p \wedge \neg q$ and $\neg p \wedge q$ are all true.
- _____ If $s_1 \Rightarrow s_2$ and $\neg s_1 \Rightarrow \neg s_2$, then $s_1 \Leftrightarrow s_2$.
- _____ The converse of $\neg p \rightarrow q$ is $\neg q \rightarrow p$.
- _____ An argument is valid if there is a truth assignment so that the premises and conclusion are all true.

2. For given integers m and n , let the statements p, q, r be defined by:

$$p : m \text{ is even}; \quad q : n \text{ is even}; \quad r : m + n \text{ is even}.$$

- (a) [1] Write the statement “if m is even and n is odd, then $m + n$ is odd” in symbolic form. (Note: these are specific integers so quantifiers are not needed.)
- (b) [1] Write the negation of the statement “if m is odd and n is odd, then $m + n$ is odd” in plain English.
- (c) [1] Write the contrapositive of the statement $(r \wedge p) \rightarrow \neg q$ in plain English.
3. [2] Find an expression logically equivalent to $\neg(p \rightarrow \neg q) \vee \neg r$ which uses only the symbols $p, q, r, \wedge, \neg$ and brackets. Give reasons for each logical equivalence used.

4. [3] Use known logical equivalences and inference rules to show the argument below is valid. Give reasons for each step.

$$\begin{array}{l} \neg c \vee s \\ \neg c \rightarrow w \\ \hline \neg w \\ \hline \therefore s \end{array}$$

5. Suppose the universe is the integers.

(a) [1] Write the statement $\forall x, \exists y, y > x$ in plain English.

(b) [1] Write the statement $\exists y, \forall x, y \geq x$ in plain English.

(c) [1] One of the statements in parts (a) and (b) is true and the other is false. Which one is false? Briefly explain why it is false.

6. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F). No reasons are necessary.

___ If x can be any real number, then the statement $\exists x, (x^2 \leq 0) \rightarrow (x = -1)$ is true.

___ The negation of “*All students like calculus*” is “*Some students like calculus*”.

___ For the universe of integers, when the statement “*if m is even, then $m \cdot n$ is even*” is written in symbols, then only universal quantifiers appear.

___ The negation of $\forall x, \exists y, x^2 < y^2$ is $\exists x, \forall y, x^2 \geq y^2$.