201709 Math 122 A01 Quiz #1

#V00:_____ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F). No reasons are necessary.

   T If $p \to q$ is false, then $p \vee q$ is true.

   F If $p \leftrightarrow q$ is true, then $p \wedge q$ is true.

   F There are truth values for p and q so that $\neg p \vee q$ and $p \vee \neg q$ are both false.

   T $(1 = 2) \to (4 > 5)$.

2. Write in English:

   (a) [1] the statement "*In order to for you complete a triathlon, it is necessary for you to know how to swim*" in the form "if p then q".

   If you can complete a triathlon then you know how to swim

   (b) [1] the statement "*In order for you to be able ride the bus, it is sufficient for you to have a U-Pass*" in the form "if p then q".

   If you have a U-Pass then you are able to ride the bus.

   (c) [1] the converse of "*if a is odd and b is odd, then a^b is odd.*"

   If a^b is odd, then a is odd and b is odd

   (d) [1] the contrapositive of "*if a is odd and b is odd, then a^b is odd.*"

   If a^b is even, then a is even or b is even

3. [2] Use any method to show that $p \to (p \vee \neg q)$ is a tautology.

   If p is false, the statement is true.
   If p is true, then so is $(p \vee \neg q)$, and
   so the statement is true.
   ∴ It is a tautology

4. Let s and g be the following statements:

   s : Gary went swimming.

   g : Gary went to the gym.

   r : It rained.

   Write each statement in symbolic form using only $s, g, r, \neg, \vee, \wedge, \rightarrow, \leftrightarrow$ and brackets.

   (a) [1] If it rained, then Gary went swimming or to the gym.

   $r \rightarrow (s \vee g)$

   (b) [1] Gary went swimming or to the gym, but not both.

   $\neg(s \leftrightarrow g)$

5. [3] Use a truth table to determine whether $\neg(\neg p \rightarrow q)$ is logically equivalent to $\neg p \wedge \neg q$.

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg p \rightarrow q$	$\neg(\neg p \rightarrow q)$
F	F	T	T	T	F	T
F	T	T	F	F	T	F
T	F	F	T	F	T	F
T	T	F	F	F	T	F
				↑		↑

These are the same

$\therefore (\neg p \wedge \neg q) \leftrightarrow \neg(\neg p \rightarrow q)$ is a tautology

$\therefore \neg p \wedge \neg q \Longleftrightarrow \neg(\neg p \rightarrow q)$

6. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F). No reasons are necessary.

   T An implication and its contrapositive always have the same truth value.

   F $p \rightarrow q$ is logically equivalent to $\neg(\neg p \wedge \neg q)$.

   T If $s_1 \wedge s_2$ is a tautology, then so is $\neg s_1 \rightarrow s_2$.

   F If s_1 is not logically equivalent to s_2, then $s_1 \leftrightarrow s_2$ is a contradiction.