201809 Math 122 A01 Quiz #1

#V00:_____ Name: Solutions

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but neither needed nor helpful. Clearly organized work must be shown when needed in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True** (T) or **False** (F). No reasons are necessary. Lower case letters represent statements.

   T If $p \wedge \neg q$ is false, then $p \to q$ is true.

   F If $\neg p \leftrightarrow \neg q$ is true, then $p \vee q$ is true.

   T An implication and its converse are not logically equivalent.

   F There are truth values for p and q so that the statements $\neg(p \leftrightarrow q)$ and $p \wedge q$ are both true.

2. Write each answer in English.

   (a) [1] Write the statement "*In order for Gary get coffee in less than 15 minutes, it is necessary that he does not go to the cafe near class-change time*" as an implication.

   IF G gets coffee in less than 15 mins, THEN he does not go to the cafe near class-change time

   (b) [3] For the statement "*If I had a million dollars, I'd buy you a fur coat*", write the

   Converse: If I'd buy you a fur coat then I have $1,000,000

   Contrapositive: If I wouldn't buy you a fur coat then I don't have $1,000,000

   Negation: I have a millim dollars and I won't buy you a fur coat

3. [2] Use any method to determine whether $p \wedge (\neg q \to p)$ is logically equivalent to p. Explain your reasoning.

   If p is false, then $p \wedge (\neg q \to p)$ is false,
   If p is true, then $\neg q \to p$ is true
   so $p \wedge (\neg q \to p)$ is true

   $\therefore$ $p \leftrightarrow (p \wedge (\neg q \to p))$ is a tautology,
   so the $p \wedge (\neg q \to p)$ is L.E. to p

4. Let m and n be integers, and let p, q, r, and s be the following statements:

p: m is odd; q: n is odd; r: $m+n$ is odd; s: mn is odd

Write each of the following in symbolic form using only $p, q, r, s, \neg, \wedge, \vee, \rightarrow, \leftrightarrow$ and brackets. Remember that any number is either even or odd (not both); the number 0 is even.

(a) [1] If mn is even, then m is even or n is even.

$\neg s \rightarrow (\neg p \vee \neg q)$

(b) [1] $m+n$ is odd if and only if m is even and n is odd, or m is odd and n is even.

$r \leftrightarrow [(\neg p \wedge q) \vee (p \wedge \neg q)]$

(c) [1] $m+n$ and mn are not both odd.

$\neg(r \wedge s)$ $\quad (\Leftrightarrow \neg r \vee \neg s)$

5. [2] Suppose that the statement $(p \vee q) \wedge (p \rightarrow \neg q)$ is false. Find all possible combinations of truth values for which the statement $(r \wedge \neg q) \vee (\neg r \wedge p)$ is true. Explain your reasoning.

Stmt false when p, q both F $(p \vee q \; F)$
& when p, q both T $(p \rightarrow \neg q \; F)$

If both p, q F, stmt true when r is T
b/c $(r \wedge \neg q)$ T & not when r is F.
If both p, q T, stmt true when r is F
b/c $(\neg r \wedge p)$ T & not when r is T.
∴ The possible combinations of truth values we want.

p	q	r
F	F	T
T	T	F

6. [2] Use the blank to indicate whether each statement is **True** (T) or **False** (F). No reasons are necessary. Lower case letters represent statements.

F The statement $p \rightarrow \neg p$ is a contradiction.

T If $s_1 \leftrightarrow s_2$ is a tautology, then $s_1 \vee \neg s_2$ is a tautology.

F The contrapositive of "*If the goods are not satisfactory, then your money is refunded and you get a store credit*" is *"If your money is not refunded and you don't get a store credit, then the goods are satisfactory"*.

F Statements which are not logically equivalent can never have the same truth value.