201909 Math 122 [A01] Quiz #1

#V00:_____ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but unnecessary! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True** (T) or **False** (F).

   T If q is false, then $p \to \neg q$ is true.

   F If $p \vee q$ is false, then $\neg p \leftrightarrow q$ is true.

   F A statement and its negation can sometimes have the same truth value.

   T There are truth values for p and q so that the statements $p \wedge \neg q$ and $\neg p \leftrightarrow q$ are both false.

2. Write each answer in English.

   (a) [1] Write the statement "*Good golfers have short memories*" in the form "if *hypothesis* then *conclusion*".

   If someone is a good golfer then they have a short memory

   (b) [3] For the statement "*If a is odd and b is odd, then $a+b$ is even*", write the

   Converse: If $a+b$ is even then a is odd and b is odd

   Contrapositive: If $a+b$ is odd then a is even or b is even

   Negation:

   a, b, and $a+b$ are all odd

3. Use the statements$p$: *Gary rides a motorcycle*, q: *Gary has good balance*, and r: *Gary has raised boots* to write each statement in symbolic form.

   (a) [1] Gary rides a motorcycle when he has good balance and raised boots.

   $(q \wedge r) \to p$

   (b) [1] In order for Gary to ride a motorcycle he needs to have raised boots.

   $p \to r$

4. [2] Show that the statements $\neg p \leftrightarrow q$ and $\neg q \rightarrow \neg p$ are not logically equivalent. Write a sentence to justify your conclusion.

If p & q are both false then $\neg p \leftrightarrow q$ is false and $\neg q \rightarrow \neg p$ is true, so that $(\neg p \leftrightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ is false. (and not a tautology) ∴ The stmts are not L.E.

5. [3] Use the Laws of Logic to show that $\neg[(p \vee q) \rightarrow q]$ is logically equivalent to $p \wedge \neg q$.

$\neg[(p \vee q) \rightarrow q)$

$\Leftrightarrow \neg[\neg(p \vee q) \vee q]$ Known L.E.

$\Leftrightarrow \neg\neg(p \vee q) \wedge \neg q$ DeMorgan & Dbl Neg'n

$\Leftrightarrow (p \wedge \neg q) \vee (q \wedge \neg q)$ Dist.

$\Leftrightarrow p \wedge \neg q$ Known contradiction & Identity

6. [2] Use the blank to indicate whether each statement is **True** (T) or **False** (F). No reasons are necessary.

T $(p \wedge q) \vee (\neg p \vee \neg q)$ is a tautology.

F $a \vee (b \wedge c)$ is logically equivalent to $(a \vee b) \wedge c$.

T If $s_1 \rightarrow s_2$ is a contradiction, then s_2 is a contradiction.

T When written in symbols, the assertion "*Exactly one of the statement p and q is true*" is $\neg(p \leftrightarrow q)$.