201609 Math 122 [A01] Quiz #1

#V00:_____ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F).

   T If q is true, then $\neg p \to q$ is true.

   F If $p \wedge q$ is false, then $p \leftrightarrow q$ is false.

   T If $s_1 \to s_2$ is a contradiction, then $s_2 \to s_1$ is a tautology.

   T There are truth values for p and q so that the three statements $p \vee q$, $\neg p \vee q$, and $p \vee \neg q$ are all true.

2. Write each answer in English.

   (a) [1] Write the statement "*Barcelona win when Suarez scores*" in the form "if p then q".

   If Suarez scores, then Barcelona win

   (b) [3] For the statement "*If n is prime, then n has exactly 2 positive divisors*", write the

   Converse: If n has exactly 2 positive divisors, then n is prime

   Negation: n is prime and does not have exactly 2 positive divisors

   Contrapositive: If n does not have exactly 2 positive divisors, then n is not prime.

3. [3] Use a truth table to determine whether $\neg p \leftrightarrow q$ is logically equivalent to $(p \vee q) \wedge (\neg p \vee \neg q)$.

					s_1	s_2	
p	q	$\neg p$	$\neg q$	$\neg p \leftrightarrow q$	$p \vee q$	$\neg q \vee \neg p$	$s_1 \wedge s_2$
F	F	T	T	F	F	T	F
F	T	T	F	T	T	T	T
T	F	F	T	T	T	T	T
T	T	F	F	F	T	F	F

Corresponding entries in the T.T. are the same

$\therefore (\neg p \leftrightarrow q) \leftrightarrow [(p \vee q) \wedge (\neg p \vee \neg q)]$ is a tautology $\therefore$ YES

4. A sign posted at a motorcycle dealership says "*In order to ride the Harley Davidson, you must have grey hair or be wearing a leather jacket. If you do not have grey hair or are not wearing a leather jacket, then you can not ride the Harley Davidson.*"

   (a) [1] Give an example of a situation where the two statements on the sign have different truth values.

   You have grey hair and are not wearing a leather jacket.
   By stmt 1 you meet the criteria to ride.
   By stmt 2 you can't.

   (b) [1] Rewrite the second statement so that it is logically equivalent to the first one.

   If you don't have grey hair and aren't wearing a leather jacket, then you can't ride the Harley Davidson.

5. [2] Suppose the universe is the integers. Determine the truth value of the statement,

$$\forall m, \exists n, mn > 0$$

and briefly justify your answer.

It is false. If $m=0$ then $mn=0$ for every n.

6. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F). No reasons are necessary.

   T If x can be any real number, then the statement $\forall x, (x^2 \leq 0) \rightarrow (x = 0)$ is true.

   F The negation of "*Every prime number is odd*" is "*No even number is prime*".

   T If "*There is a largest integer*" is written in symbols, the quantifiers $\forall$ and $\exists$ are both used.

   F If the universe is the integers, then $\forall x, \exists y, x^2 < y^2$ and $\exists y, \forall x, x^2 < y^2$ have the same truth value.