

201909 Math 122 [A01] Quiz #2

#V00: _____

Name: _____

This quiz has 2 pages and 7 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but unnecessary! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**.

___ If s_1 logically implies s_2 , and s_2 logically implies s_1 , then s_1 and s_2 are logically equivalent.

___ An argument can be proved to be valid by showing that the negation of the conclusion logically implies some premise is false.

___ The conclusion of a valid argument can be false.

___ $\neg(p \leftrightarrow q)$ logically implies $p \vee q$.

2. [2] Show that the argument below is invalid by giving a counterexample, and writing a sentence to justify your conclusion.

$$\frac{a \vee b}{c \leftrightarrow \neg(a \vee b)} \\ \therefore \neg a \vee c$$

3. [2] Find a statement logically equivalent to $r \vee (p \leftrightarrow q)$ which uses only the symbols p, q, r, \wedge, \neg and brackets.

4. [1] Suppose the universe is the integers. Determine the truth value of the statement $\exists x, \forall y, xy \leq 0$, and briefly explain your reasoning.

5. [4] Write the argument below in symbolic form, and then use known logical equivalences and inference rules to show it is valid. Remember to define the letters you use to represent statements.

If I did not make a quiz, then I need to work late
 If I am not behind at work, then I can go shopping
 The statement that I need to work late or am behind at work is false

∴ I made a quiz and could go shopping

6. [2] Let n be an integer. Prove that *if n is even, then $3n$ is even*.

7. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

___ If $\neg p$ logically implies a contradiction, then $p \rightarrow q$ is a tautology.

___ For the universe of real numbers, $\exists x, \forall y, (y \neq 0) \rightarrow (xy = 1)$.

___ When the statement “*if m is odd and n is even, then $m + n$ is odd*” is written in symbols, two universal quantifiers and three existential quantifiers appear.

___ $\forall x, \neg(p(x) \vee q(x))$ is logically equivalent to $\exists x, \neg p(x) \wedge \neg q(x)$.