202409 Quiz 2
Math 122 A04
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Solutions

Do not open the booklet before you are told to

Date and Time: Tuesday, October 1, 2024 at 1:55pm.

Instructions: There are 3 pages and 5 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the sheet for rough work.

True/False Instructions: Question 1 consists of 8 true/false questions labelled **TF 1** to **TF 8**. The last page of your test booklet is a bubble sheet for answering them. You may detach the back page from the rest of the test if you wish. Only fill in a bubble for questions 1-8 on the bubble sheet. When making your selection, **True is A** and **False is B**. Do not select C, D or E. If you want to change your answer after filling in a bubble, then please erase your previous answer or write something on the sheet to try to make your final selection as clear as possible.

1. [4] Use the bubble sheet provided on the last page of the test booklet to indicate whether each statement is **True (A)** or **False (B)**.

F **[TF 1]** $p \vee \mathbf{1} \Leftrightarrow p \wedge \mathbf{0}$.

T **[TF 2]** If $s_1 \Rightarrow s_2$, then $s_1 \Rightarrow (s_1 \wedge s_2)$.

F **[TF 3]** The statement p is logically equivalent to $p \leftrightarrow p$.

F **[TF 4]** If an argument is valid, then all of the premises are always true.

F **[TF 5]** For the universe of the integers, what is the truth value of $\exists n, \exists m, n + m = \frac{1}{2}$?

F **[TF 6]** For the universe of the real numbers, what is the truth value of $\forall x, \exists y, xy = 1$?

F **[TF 7]** The negation of the statement "*Some cars are red*" is "*Some cars are not red*".

T **[TF 8]** The negation of $\forall x, \exists y, x + y > 1$ is $\exists x, \forall y, x + y \leq 1$.

2. [3] Use Laws of Logic and Known Logical Equivalences to show that $\neg((p \to q) \vee (q \to \neg p))$ is a contradiction.

$\neg((p \to q) \vee (q \to \neg p))$

$\Leftrightarrow \neg((\neg p \vee q) \vee (\neg q \vee \neg p))$, Implication (x2)

$\Leftrightarrow \neg((\neg p \vee \neg p) \vee (q \vee \neg q))$, Associativity Commutativity

$\Leftrightarrow \neg((\neg p \vee \neg p) \vee \mathbf{1})$, Known tautology

$\Leftrightarrow \neg(\mathbf{1})$, Dominance

$\Leftrightarrow \mathbf{0}$, Known LE.

3. [2] Show that the argument below is invalid by giving a counterexample. Write a sentence to explain why your counterexample shows that the argument is invalid.

$$\begin{array}{ll} & d \vee a \\ & c \to \neg b \\ \hline \therefore & a \vee (a \leftrightarrow b) \end{array}$$

To make the conclusion false we need a false and $a \leftrightarrow b$ false which, since a is false, means b must be true. To make the premise $d \vee a$ true we need d to be true and to make $c \to \neg b$ true we need c to be false.

$\begin{pmatrix} a & b & c & d \\ 0 & 1 & 0 & 1 \end{pmatrix}$ is a counterexample because the premises are all true and the conclusion is false.

4. [3] Use Laws of Logic, Known Logical Equivalences and Inference Rules to show that the following argument is valid.

$$\begin{array}{ll} & \neg(p \to q) \\ & \neg q \to r \\ & \neg s \to \neg r \\ \hline \therefore & p \wedge s \end{array}$$

1. $\neg(p \to q)$, Premise
2. $\neg(\neg p \vee q)$ 1, Implication
3. $p \wedge \neg q$, 2. DeMorgan, Double Neg.
4. p, 3, Conjunctive Simplification
5. $\neg q \to r$, Premise
6. $\neg s \to \neg r$, Premise
7. $r \to s$, Contrapositive, Double Neg (x2)
8. $\neg q \to s$, 5, 7, Chain rule
9. $\neg q$, 3, Conjunctive Simplification
10. s, 8, 9, Modus Ponens
11. $p \wedge s$, 4, 10

Therefore the argument is valid.

5. [3] Let n and m be integers. Prove that, if $n + 2m$ is even, then n is even. (Hint: Contrapositive).

Proof

~~Suppose that~~ We prove the contrapositive. That is, we prove that, if n is odd, then $n+2m$ is odd.

Suppose n is odd. Then $\exists$ an integer k such that $n = 2k+1$. So,

$$n+2m = 2k+1+2m = 2(k+m)+1.$$

Since $k+m$ is an integer, $n+2m$ is odd.

$\square$