

202401 Math 122 A01 Quiz #2

#V _____

Name: Key

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a Sharp calculator with model number beginning EL510R. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Let $A = \{1, 2, \{1, 2\}\}$. Use the blank to indicate whether each statement is true (T) or false (F). No reasons are necessary.

T $|A| = 3.$

T $\emptyset \subsetneq A.$

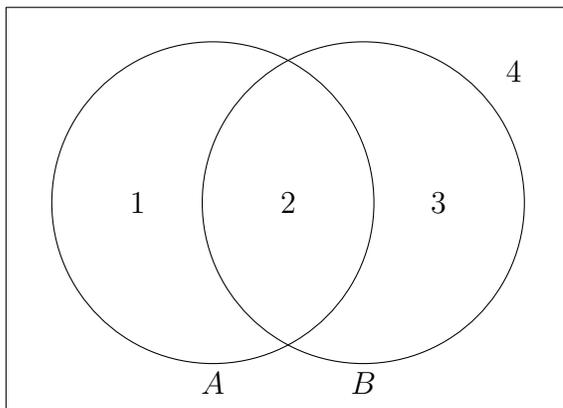
T $\{1, 2\} \in A \cap \mathcal{P}(A).$

F There are no subsets of $\emptyset.$

2. [3] Use the Laws of Set Theory to prove that, for all sets A and B , $(A^c \setminus B)^c = A \cup B$. Give reasons for each step.

$$\begin{aligned}
 (A^c \setminus B)^c &= (A^c \cap B^c)^c && \text{Known} \\
 &= A^{cc} \cup B^{cc} && \text{De Morgan +} \\
 &= A \cup B && \text{Dbl Comp.}
 \end{aligned}$$

3. [2] Use the Venn Diagram below to help describe a universe and sets A and B which demonstrate that the statement $A^c \setminus B = (A \oplus B)^c$ is not true for all sets A and B .



$$U = \{1, 2, 3, 4\}$$

$$A = \{1, 2\}, B = \{2, 3\}$$

$$A^c = \{3, 4\}$$

$$\underline{A^c \setminus B = \{4\}}$$

$$A \oplus B = \{1, 3\}$$

$$\underline{(A \oplus B)^c = \{2, 4\}}$$

$\therefore A^c \setminus B$ is not equal to $(A \oplus B)^c$ for all sets A, B .

4. (a) [3] Let A and B be sets such that $A \cap B = A$. Prove that $A \subseteq B$.

Take any $x \in A$.

Since $A = A \cap B$, $x \in A \cap B$.

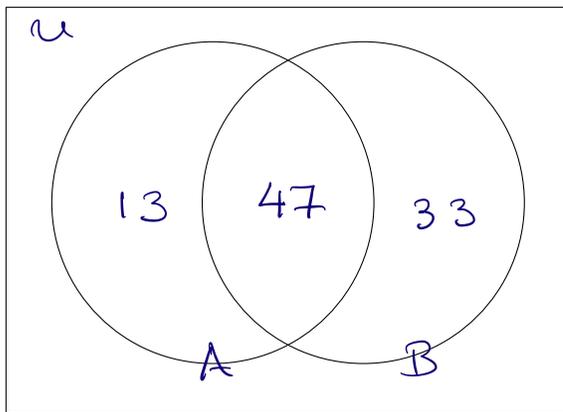
If $x \in A \cap B$, then $x \in B$ by definition of intersection.

$\therefore A \subseteq B$

- (b) [1] Give an example to show that the sets A and B in (a) need not be equal. Use the universe $U = \{1, 2\}$.

Let $A = \emptyset$ and $B = \{1\}$. Then $A \cap B = \emptyset = A$, $A \subseteq B$, and $A \neq B$.

5. In a group of 150 students, 60 are taking Math 122, 80 are taking Math 101, and 47 are taking both of these classes. Answer (a) and (b) below. If you fill in the Venn Diagram below, remember to define any sets used.



Let $A =$ set of students taking 122
 $\& B =$ set of students taking 101

- (a) [1] How many students are taking neither Math 122 nor Math 101?

$$= 150 - |A \cup B| = 150 - 93 = 57$$

- (b) [1] How many students are taking only Math 122?

$$= |A \setminus B| = 13$$

6. [2] Use the blank to indicate whether each statement is true (T) or false (F). No reasons are necessary.

T $A \cap B \subseteq B$ for all sets A and B .

T If A and B are sets and $A \subsetneq B$, then $A \oplus B \neq \emptyset$.

T If $A = \{1, 2\}$, then A has exactly 2 non-empty proper subsets.

F The number of subsets of $\{1, 2, \dots, 10\}$ that contain 1 and not 2 equals 2^9 .