202201 Math 122 A01 Quiz #2

#V00:________________ Name: ____________________________

This quiz has 2 pages and 7 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but are neither needed nor helpful. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   ___ If statements s_1 and s_2 are logically equivalent, then s_1 logically implies s_2 and $\neg s_1$ logically implies $\neg s_2$.

   ___ The conclusion of a valid argument is a tautology.

   ___ Any statement is logically equivalent to a statement that uses only the logical connective $\wedge$ and negation.

   ___ The contrapositive of the open statement $p(x) \to q(x)$ is $\neg q(x) \to \neg p(x)$.

2. [3] Use known logical equivalences and the Laws of Logic to show that $\neg b \wedge (a \to b)$ is logically equivalent to $\neg(a \vee b)$. Give reasons for each step.

3. [2] Give a counterexample to show that the following argument is invalid. Write a sentence to explain your conclusion.

$$\begin{array}{rr} & a \to \neg b \\ & b \vee c \\ & \neg a \\ \hline \therefore & c \end{array}$$

4. [3] Use known logical equivalences and inference rules to show that the argument below is valid.

$$\begin{array}{rl} & \neg(p \to q) \\ & \neg r \leftrightarrow q \\ \hline \therefore & r \end{array}$$

5. [1] Let the universe consist of the numbers 0, 1, and 2. Write a statement in symbols, without quantifiers, which is logically equivalent to $\forall x, (2x + 1 = 5) \to (x^3 = 8)$.

6. [2] For the universe $\mathbb{Z}$, let $p(x)$ be the statement "x *is odd*". Write the the statement $\exists m, \exists n, \neg p(mn) \wedge p(m + n)$ in plain English, and determine its truth value.

7. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

___ For a given universe both $\forall x, p(x)$ and $\exists x, \neg p(x)$ can be true.

___ If $\exists x, p(x)$ is true and $\exists x, q(x)$ is true, then $\exists x, p(x) \wedge q(x)$ is true.

___ When the statement "*any statement is logically equivalent to a statement that uses only the logical connective* $\wedge$ *and negation*" is written in symbols, both a universal and an existential quantifier appear.

___ For the universe $\mathbb{R}$, $\neg\exists x, \forall y, x < y$ is logically equivalent to $\forall x, \exists y, x \geq y$.