202201 Math 122 A01 Quiz #2

#V00:________________ Name: Key

This quiz has 2 pages and 7 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but are neither needed nor helpful. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   T If statements s_1 and s_2 are logically equivalent, then s_1 logically implies s_2 and $\neg s_1$ logically implies $\neg s_2$.

   F The conclusion of a valid argument is a tautology.

   T Any statement is logically equivalent to a statement that uses only the logical connective $\wedge$ and negation.

   T The contrapositive of the open statement $p(x) \to q(x)$ is $\neg q(x) \to \neg p(x)$.

2. [3] Use known logical equivalences and the Laws of Logic to show that $\neg b \wedge (a \to b)$ is logically equivalent to $\neg(a \vee b)$. Give reasons for each step.

$\neg b \wedge (a \to b)$

$\Leftrightarrow \neg b \wedge (\neg a \vee b)$ Known

$\Leftrightarrow (\neg b \wedge \neg a) \vee \cancel{(\neg b \wedge b)}^{\,0}$ Dist.

$\Leftrightarrow \neg b \wedge \neg a$ Known contradiction, Identity

$\Leftrightarrow \neg(a \vee b)$ DeMorgan

3. [2] Give a counterexample to show that the following argument is invalid. Write a sentence to explain your conclusion.

$$\begin{array}{rl} a \to \neg b & T \\ b \vee c & T \\ \neg a & T \\ \hline \therefore \quad c & F \end{array}$$

For the truth assignment $\begin{pmatrix} a & b & c \\ F & T & F \end{pmatrix}$ all premises are true & the conclusion is false.

$\therefore$, The argument is invalid.

4. [3] Use known logical equivalences and inference rules to show that the argument below is valid.

$$\begin{array}{rl} & \neg(p \to q) \\ & \neg r \leftrightarrow q \\ \hline \therefore & r \end{array}$$

1.	$\neg(p \to q)$	Premise
2.	$\neg r \leftrightarrow q$	"
3.	$(\neg r \to q) \wedge (q \to \neg r)$	2, L.E.
4.	$\neg r \to q$	3, Conj Simp.
5.	$\neg q \to r$	4, Contrapos
6.	$\neg(\neg p \vee q)$	1, L.E.
7.	$p \wedge \neg q$	6, DeMorgan
8.	$\neg q$	7, Conj Simp
9.	$\therefore r$	5, 8, M.P.

5. [1] Let the universe consist of the numbers 0, 1, and 2. Write a statement in symbols, without quantifiers, which is logically equivalent to $\forall x, (2x + 1 = 5) \to (x^3 = 8)$.

$$[(2\cdot 0=5) \to (0^3=8)]$$
$$\wedge\ [(2\cdot 1=5) \to (1^3=8)]$$
$$\wedge\ [(2\cdot 2=5) \to (2^3=8)]$$

6. [2] For the universe $\mathbb{Z}$, let $p(x)$ be the statement "*x is odd*". Write the the statement $\exists m, \exists n, \neg p(mn) \wedge p(m + n)$ in plain English, and determine its truth value.

There are integers m and n such that mn is even and m+n is odd.

The statement is true. Take m=0 and n=1. Then mn=0, which is even and m+n=1, which is odd.

7. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

F For a given universe both $\forall x, p(x)$ and $\exists x, \neg p(x)$ can be true.

F If $\exists x, p(x)$ is true and $\exists x, q(x)$ is true, then $\exists x, p(x) \wedge q(x)$ is true.

T When the statement "*any statement is logically equivalent to a statement that uses only the logical connective $\wedge$ and negation*" is written in symbols, both a universal and an existential quantifier appear.

T For the universe $\mathbb{R}$, $\neg\exists x, \forall y, x < y$ is logically equivalent to $\forall x, \exists y, x \geq y$.