201701 Math 122 [A01] Quiz #2

#V00:________________ Name: Solutions

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   T The sentence "*Every theorem has a proof*" contains a hidden existential quantifier.

   F The negation of $\exists y, \exists x, xy \neq 0$ is $\forall y, \exists x, xy \neq 0$ (assume the universe is the integers).

   F Two statements can be proved to be logically equivalent by showing that they can have the same truth values.

   T If the conclusion of an argument is guaranteed to be true, then the argument is valid.

2. [2] Suppose the universe is the real numbers. Briefly explain why the statement

$$\forall x, \exists y, xy = 1$$

is false. Consider $x=0$. Then $xy = 0y = 0$ for any y. Thus, when $x=0$, there is no y such that $xy=1$. $\therefore$ $\exists y, xy=1$ is not true for all x.

3. [2] Use known logical equivalences to show that $[p \vee (q \vee p)] \wedge (q \to p)$ is logically equivalent to p.

$[p \vee (q \vee p)] \wedge (q \to p)$

$\Leftrightarrow [p \vee (p \vee q)] \wedge (\neg q \vee p)$ Comm, Known LE

$\Leftrightarrow [(p \vee p) \vee q] \wedge (p \vee \neg q)$ Assoc, Comm

$\Leftrightarrow [p \vee q] \wedge (p \vee \neg q)$ Idempotent

$\Leftrightarrow p \vee \underbrace{(q \wedge \neg q)}_{0}$ Dist've

$\Leftrightarrow p \vee 0$ Known contradiction

$\Leftrightarrow p$ Identity

4. [2] Find an expression which is logically equivalent to $p \to \neg q$ and involves only the symbols $p, q, \wedge, \neg$ and brackets.

$p \to \neg q \iff \neg p \vee \neg q$ Known LE

$\iff \neg(p \wedge q)$ DeMorgan

5. [3] Give a counterexample to show that the following argument is not valid, and briefly explain your conclusion.

$$\begin{array}{rl} a \vee b & 1 \\ \neg b \leftrightarrow c & 1 \\ \hline \therefore \quad a \to b & 0 \end{array} \qquad \begin{pmatrix} a & b & c \\ 1 & 0 & 1 \end{pmatrix}$$

The given T.A. is such that the conclusion is false and all premises are true, therefore the argument is not valid.

6. [4] Write the argument below in symbolic form, and then use known logical equivalences and inference rules to show that it is valid. Remember to clearly and explicitly define the letters you use to represent statements.

I went cycling or I sat on the couch watching football
If I corrected my paper, then I did not sit on the couch watching football
I did not go cycling
∴ I did not correct my paper

Let c be "I went cycling",
f " "I sat on the couch watching football"
p " "I corrected my paper"

The argument is:

$$\begin{array}{l} c \vee f \\ p \to \neg f \\ \neg c \\ \hline \therefore \neg p \end{array}$$

Proof:

1. $c \vee f$ Premise
2. $p \to \neg f$ "
3. $\neg c$ "
4. $\neg c \to f$ L.E. to 1
5. $f \to \neg p$ Contrapos of 2
6. $\neg c \to \neg p$ 4,5 Chain Rule
7. $\neg p$ 3,6 M.P.