201709 Math 122 A01 Quiz #2

#V00:__________________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F). No reasons are necessary.

   T If two statements are logically equivalent, then so are their negations.

   T If the conclusion of a valid argument is false, then one of the premises must be false.

   T If $p \Rightarrow q$ and $\neg p \Rightarrow \neg q$, then p and q are logically equivalent.

   T $(p \vee q) \wedge \neg p$ logically implies q

2. [3] Use known logical equivalences to show that $\neg(\neg a \to b) \vee b$ is logically equivalent to $a \to b$.

$\neg(\neg a \to b) \vee b$

$\Leftrightarrow \neg(a \vee b) \vee b$ Known LE

$\Leftrightarrow (\neg a \wedge \neg b) \vee b$ 1 DeMorgan

$\Leftrightarrow (\neg a \vee b) \wedge \cancel{(\neg b \vee b)}$ Distributive

$\Leftrightarrow \neg a \vee b$ Identity, known tautology.

$\Leftrightarrow a \to b$ Known LE.

3. [2] Find an expression logically equivalent to $(a \to b) \to c$ that uses only the symbols $a, b, c, \neg, \wedge$ and brackets.

$(a \to b) \to c$

$\Leftrightarrow (\neg a \vee b) \to c$ Known LE

$\Leftrightarrow \neg(\neg a \vee b) \vee c$ Known LE.

$\Leftrightarrow (a \wedge \neg b) \vee c$ DeMorgan

$\Leftrightarrow \neg(\neg(a \wedge \neg b) \wedge \neg c)$ "

4. [3] Use known logical equivalences and inference rules to show that the following argument is valid.

$$\begin{array}{ll} & \neg b \\ & a \leftrightarrow \neg b \\ & c \to \neg a \\ \hline \therefore & \neg c \end{array}$$

1. $\neg b$	Premise
2. $a \longleftrightarrow \neg b$	"
3. $c \longrightarrow \neg a$	"
4. $(a \to \neg b) \wedge (\neg b \to a)$	2, L.E.
5. $\neg b \to a$	4, Conj. Simp.
6. $\therefore a$	1, 5, M.P.
7. $a \longrightarrow \neg c$	3, Contrapos.
8. $\therefore \neg c$	6, 7 M.P.

5. [3] Write the argument below in symbolic form, and then give a counterexample to show that it is not valid. Remember to define the letters you use to represent statements.

If I watch golf, then I think about mathematics
If I do not think mathematics, then I watch football
∴ If I don't watch football, then I watch golf

Notation
p: I watch golf
q: I think about math
r: I watch football

Argument

$$\begin{array}{l} p \to q \\ \neg q \to r \\ \hline \therefore \neg r \to p \end{array}$$

For the truth ass'nt $\begin{pmatrix} p & q & r \\ 0 & 1 & 0 \end{pmatrix}$, all premises are T & the conclusion is F.
∴ argument invalid.

6. [2] Use the blank to indicate whether each statement is **true** (T) or **false** (F). No reasons are necessary.

F The converse of $\exists x, (x^2 > 0) \to (x \neq 0)$ is $\forall x, (x = 0) \to (x^2 \leq 0)$.

T The negation of "*All Sedin penalties are hooking or holding*" is "*Some Sedin penalties are ~~neither~~ neither hooking nor holding*". OOPS!

F $\exists x, \forall y, x + y = 1$, where the universe is the integers.

F When "*A circle can be inscribed in any triangle*" is written in symbols, exactly one quantifier appears.