201809 Math 122 A01 Quiz #2

#V00:_____ Name: Solutions

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but are neither needed nor helpful. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary. Lower case letters denote statements.

   T The statements $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

   F The Associative Law states that $p \wedge (q \vee r)$ is logically equivalent to $(p \wedge q) \vee r$.

   T If statements s_1 and s_2 are logically equivalent, then s_1 logically implies s_2.

   F If the premises of an argument can never be true, then the argument is invalid.

2. [3] Use known logical equivalences to show that $(p \vee q) \wedge \neg(\neg p \wedge q)$ is logically equivalent to p.

$(p \vee q) \wedge \neg(\neg p \wedge q)$

$\Leftrightarrow (p \vee q) \wedge (p \vee \neg q)$ DeMorgan, Dbl Neg'n.

$\Leftrightarrow p \vee (q \wedge \neg q)$ Dist

$\Leftrightarrow p \vee 0$ Known $\Rightarrow\Leftarrow$

$\Leftrightarrow p$ Identity

3. [2] Find an expression which is logically equivalent to $p \leftrightarrow \neg q$ and involves only the symbols $p, q, \wedge, \neg$ and brackets.

$p \leftrightarrow \neg q$

$\Leftrightarrow (p \to \neg q) \wedge (\neg q \to p)$ Known LE

$\Leftrightarrow (\neg p \vee \neg q) \wedge (q \vee p)$ Known LE

$\Leftrightarrow \neg(p \wedge q) \wedge \neg(\neg q \wedge \neg p)$ DeMorgan Dbl Neg'n.

4. [3] Give a counterexample to show that the following argument is invalid. Briefly explain your reasoning.

$$\begin{array}{l} \neg a \vee b \\ \neg b \to \neg c \\ \hline \therefore a \to c \end{array}$$

A Counterexample is a truth assignment s.t. the premises are true & the conclusion is false. The T.A. $\begin{pmatrix} a & b & c \\ T & T & F \end{pmatrix}$ does just that. ∴ the argument is invalid.

5. [3] Use known logical equivalences and inference rules to show that the argument below is valid.

$$\begin{array}{l} \neg c \vee \neg f \\ \neg p \to f \\ c \\ \hline \therefore p \end{array}$$

1. $\neg c \vee \neg f$	Premise
2. $\neg p \to f$	"
3. c	"
4. $c \to \neg f$	1, I.E.
5. $\neg f \to p$	2, Contrapos
6. $c \to p$	4, 5 Chain Rule
7. p	3, 6 M.P.

6. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

T For each element x in the universe, the open statements $\neg[p(x) \vee q(x)]$ and $\neg p(x) \wedge \neg q(x)$ have the same truth value.

T For a given universe which contains at least one item and open statement $p(x)$, if $\forall x, p(x)$ is true, then $\exists x, p(x)$ is true.

T For the universe of integers, what is the truth value of $\exists x, (x^2 = 1) \leftrightarrow (x = -1)$?

F For the universe of integers, what is the truth value of $\forall n, 2n > 3n$?