201909 Math 122 [A01] Quiz #2

#V00:________________ Name: Key

This quiz has 2 pages and 7 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but unnecessary! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**.

   T If s_1 logically implies s_2, and s_2 logically implies s_1, then s_1 and s_2 are logically equivalent.

   T An argument can be proved to be valid by showing that the negation of the conclusion logically implies some premise is false.

   T The conclusion of a valid argument can be false.

   T $\neg(p \leftrightarrow q)$ logically implies $p \vee q$.

2. [2] Show that the argument below is invalid by giving a counterexample, and writing a sentence to justify your conclusion.

$$\begin{array}{l} a \vee b \\ c \leftrightarrow \neg(a \vee b) \\ \hline \therefore \quad \neg a \vee c \quad F \end{array}$$

$$\begin{pmatrix} a & b & c \\ T & T & F \end{pmatrix}$$
or F

The given T.A. makes all premises true and the conclusion false.
$\therefore$ The argument is invalid

3. [2] Find a statement logically equivalent to $r \vee (p \leftrightarrow q)$ which uses only the symbols $p, q, r, \wedge, \neg$ and brackets.

$$r \vee (p \leftrightarrow q) \Leftrightarrow r \vee [(p \to q) \wedge (q \to p)] \quad \text{Known}$$
$$\Leftrightarrow r \vee [(\neg p \vee q) \wedge (\neg q \vee p)] \quad \text{Known}$$
$$\Leftrightarrow r \vee [\neg(p \wedge \neg q) \wedge \neg(q \wedge \neg p)] \quad \text{De M.}$$
$$\Leftrightarrow \neg[\neg r \wedge \neg[\neg(p \wedge \neg q) \wedge \neg(q \wedge \neg p)]] \quad ''$$

4. [1] Suppose the universe is the integers. Determine the truth value of the statement $\exists x, \forall y, xy \leq 0$, and briefly explain your reasoning.

True. When $x = 0$ we have $xy = 0 \cdot y = 0 \leq 0$ for every integer y.

5. [4] Write the argument below in symbolic form, and then use known logical equivalences and inference rules to show it is valid. Remember to define the letters you use to represent statements.

   If I did not make a quiz, then I need to work late
   If I am not behind at work, then I can go shopping
   The statement that I need to work late or am behind at work is false
   ∴ I made a quiz and could go shopping

Let m : I made a quiz
l : I need to work late
b : I am behind at work
s : I can go shopping

Then the argument is:
$\neg m \to l$
$\neg b \to s$
$\neg(l \vee b)$
$\therefore m \wedge s$

1.	$\neg m \to l$	Premise
2.	$\neg b \to s$	"
3.	$\neg(l \vee b)$	"
4.	$\neg l \wedge \neg b$	3, DeM
5.	$\neg b$	4. Conj Smp
6.	$\therefore s$	2, 5, M.P.
7.	$\neg l \to m$	1, Contrapos
8.	$\neg l$	4, Conj. Smp
9.	$\therefore m$	7, 8, M.P.
10.	$s \wedge m$	6, 9 Conjunction

∴ The argument is valid.

6. [2] Let n be an integer. Prove that *if n is even, then $3n$ is even.*

Suppose n is even.
Then there exists an integer k s.t. $n = 2k$.
$\therefore 3n = 3(2k) = 2\cdot(3k)$
Since $3k$ is an integer, $3n$ is even

7. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

F If $\neg p$ logically implies a contradiction, then $p \to q$ is a tautology.

F For the universe of real numbers, $\exists x, \forall y, (y \neq 0) \to (xy = 1)$.

T When the statement "*if m is odd and n is even, then $m + n$ is odd*" is written in symbols, two universal quantifiers and three existential quantifiers appear.

F $\forall x, \neg(p(x) \vee q(x))$ is logically equivalent to $\exists x, \neg p(x) \wedge \neg q(x)$.