201609 Math 122 [A03] Quiz #2

#V00:_____ Name: Key

This quiz has 2 pages and 5 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   T If $\forall x, s(x)$ is false, then $\exists x, \neg s(x)$ is true.

   T If s_1 logically implies s_2, and $\neg s_1$ logically implies $\neg s_2$, then s_1 and s_2 are logically equivalent.

   T If $\neg p$ logically implies a contradiction, then p is true.

   F An argument can be proved to be valid by giving an example where all of the premises are true, and so is the conclusion.

2. [3] Use known logical equivalences to show that $(p \wedge q) \vee (\neg p \wedge \neg q)$ is logically equivalent to $(p \vee \neg q) \wedge (\neg p \vee q)$.

$(p \wedge q) \vee (\neg p \wedge \neg q)$

$\Leftrightarrow ((p \wedge q) \vee \neg p) \wedge ((p \wedge q) \vee \neg q)$ Dist.

$\Leftrightarrow [(p \vee \neg p) \wedge (q \vee \neg p)] \wedge [(p \vee \neg q) \wedge (q \vee \neg q)]$ Dist.

$\Leftrightarrow (q \vee \neg p) \wedge (p \vee \neg q)$ Known tautologies & Identity

$\Leftrightarrow (p \vee \neg q) \wedge (\neg p \vee q)$ Comm x 2.

3. [3] Give a counterexample to show that the following argument is not valid.

b	1
$\neg a \leftrightarrow b$	1
$\neg c \rightarrow a$	1
$\therefore\ c \rightarrow \neg b$	0

For the truth assignment

a	b	c
0	1	1

all premises are true and the conclusion is false

$\therefore$ The argument is not valid.

4. [3] Find an expression logically equivalent to $(p \to \neg q) \to \neg r$ which uses only the symbols $p, q, r, \wedge, \neg$ and brackets.

$(p \to \neg q) \to \neg r$

$\Leftrightarrow (\neg p \vee \neg q) \to \neg r$ Known LE

$\Leftrightarrow \neg(\neg p \vee \neg q) \vee \neg r$ Known LE

$\Leftrightarrow (p \wedge q) \vee \neg r$ DeMorgan

$\Leftrightarrow \neg(\neg(p \wedge q) \wedge r)$ DeMorgan, Dbl Negn.

5. [4] Write the argument below in symbolic form, and then use known logical equivalences and inference rules to show that it is valid. Remember to define the letters you use to represent statements.

If I went cycling, then I watched sports on tv
If I didn't go cycling, then I went walking
I did not go walking

$\therefore$ I went cycling and watched sports on tv

Let C: I went cycling
S: I watched sports on tv
W: I went walking

The argument is

$C \to S$
$\neg C \to W$
$\neg W$
$\therefore C \wedge S$

1. $C \to S$	Premise
2. $\neg C \to W$	"
3. $\neg W$	"
4. $\neg W \to C$	2. Contrapos.
5. C	3, 4 M.P.
6. S	1, 5 M.P.
7. $C \wedge S$	5, 6 Conjunction

$\therefore$ The argument is valid.