201801 Math 122 A01 Quiz #2

#V00:_______________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **true (T)** or **false (F)**. No reasons are necessary. All letters represent statements.

   T If $p \leftrightarrow q$ is a tautology, then q logically implies p.

   F The conclusion of a valid argument can never be false.

   T It is possible for p to logically imply q but not be logically equivalent to it.

   T If s is a contradiction, then s logically implies q for any statement q.

2. [3] Use known logical equivalences to show that $\neg[\neg(p \vee q) \vee (\neg q \wedge p)]$ is logically equivalent to q.

$\neg[\neg(p \vee q) \vee (\neg q \wedge p)]$

$\Leftrightarrow \cancel{\neg\neg}(p \vee q) \wedge \neg(\neg q \wedge p)$ DeM, dbl neg'n

$\Leftrightarrow (p \vee q) \wedge (q \vee \neg p)$ " "

$\Leftrightarrow (p \vee q) \wedge (\neg p \vee q)$ Comm

$\Leftrightarrow \underbrace{(p \wedge \neg p)}_{F} \vee q$ Dist

$\Leftrightarrow q$ Identity

3. [2] Give a counterexample to show that the following argument is not valid. Explain why your counterexample works.

$$\begin{array}{rl} & \neg(a \wedge b) \\ & a \leftrightarrow \neg b \\ & c \rightarrow a \\ \hline \therefore & c \rightarrow b \end{array}$$

For the T.A. $\begin{pmatrix} a & b & c \\ 1 & 0 & 1 \end{pmatrix}$ all premises are true and the conclusion is false $\therefore$ the argument is invalid.

4. [3] Find an expression logically equivalent to $(\neg p \to q) \to r$ which uses only the symbols $p, q, r, \wedge, \neg$ and brackets.

$(\neg p \to q) \to r \iff \neg(\cancel{\neg\neg} p \vee q) \vee r$ Known LE Dbl neg'n

$\iff (\neg p \wedge \neg q) \vee r$ DeM.

$\iff \neg(\neg(\neg p \wedge \neg q) \wedge \neg r)$ ''

5. [3] Write the argument below in symbolic form, and then use known logical equivalences and inference rules to show that it is valid. Remember to define the letters you use to represent statements.

If I went hiking, then I did not watch football
If I didn't go hiking, then I went cycling
I did not go cycling

∴ I went hiking and didn't watch football

Let h: I went hiking
f: I watched football
c: I went cycling

The argument is

$h \to \neg f$
$\neg h \to c$
$\neg c$

$\therefore h \wedge \neg f$

1. $h \to \neg f$	premise
2. $\neg h \to c$	"
3. $\neg c$	"
4. $\neg c \to h$	2, c'pos
5. $\therefore h$	3,4 MP.
6. $\therefore \neg f$	1,5 MP
7. $h \wedge \neg f$	5,6 conjunction

6. [2] Use the blank to indicate whether each statement is **true (T)** or **false (F)**. No reasons are necessary.

T If $p(x)$ and $q(x)$ are open statements, where x comes from some universe, then $\forall x, \neg p(x) \vee q(x)$ is logically equivalent to $\forall x, p(x) \to q(x)$.

T For the universe of integers, $\exists x, (x < 0) \to (x = 4)$.

F For the universe of real numbers, $\forall x, \exists y, xy \neq 0$.

F For the universe of integers, $\forall x, \exists y, x^2 < y^2$ and $\exists y, \forall x, x^2 < y^2$ have the same truth value.