Answer key

202409 Quiz 3
Math 122 A04
Instructor: Jonathan Noel

First name (please write as legibly as possible within the boxes)

Last name

Student ID number

Do not open the booklet before you are told to

Date and Time: Tuesday, October 15, 2024 at 1:55pm.

Instructions: There are 3 pages and 5 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the sheet for rough work.

True/False Instructions: Question 1 consists of 8 true/false questions labelled **TF 1** to **TF 8**. The last page of your test booklet is a bubble sheet for answering them. You may detach the back page from the rest of the test if you wish. Only fill in a bubble for questions 1-8 on the bubble sheet. When making your selection, **True is A** and **False is B**. Do not select C, D or E. If you want to change your answer after filling in a bubble, then please erase your previous answer or write something on the sheet to try to make your final selection as clear as possible.

Do your rough work here. This will not be marked.

1. [4] Use the bubble sheet provided on the last page of the test booklet to indicate whether each statement is **True (A)** or **False (B)** for all finite sets A and B.

T [**TF 1**] If $A \subseteq B$ and $|A| < |B|$, then $A \subsetneq B$.

F [**TF 2**] $|\mathcal{P}(A^c)| = 2^{|A|}$.

T [**TF 3**] If $x \in A$, then $A \setminus \{x\} \in \mathcal{P}(A)$.

T [**TF 4**] If $A \subsetneq B$, then $A \cap B = A$.

T [**TF 5**] $\{x \in \mathbb{Z} : 2x + 5 = 0\} \subseteq \mathbb{N}$.

T [**TF 6**] $|\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}| = 3$.

T [**TF 7**] $\sqrt{2} \in \mathbb{R}$.

F [**TF 8**] $\{\frac{1}{2}, 4\} \in \mathbb{Q}$.

2. [3] Let A and B be sets. Prove that if $A \subseteq B^c$, then $A \subseteq A \setminus B$.

Suppose that $A \subseteq B^c$. Recall that $A \setminus B = A \cap B^c$. Let $x \in A$. Then, since $A \subseteq B^c$, we know that $x \in B^c$. So, $x \in A$ and $x \in B^c$ which tells us that $x \in A \cap B^c = A \setminus B$. □

3. [2] Let A, B and C be sets. Give a counterexample to demonstrate that the following statement is false and briefly explain your reasoning: *if* $A \subseteq B \cup C$, *then* $A \subseteq B$ *or* $A \subseteq C$.

Let $A = \{1, 2, 3\}$, $B = \{1, 2\}$ and $C = \{1, 3\}$. Then $B \cup C = \{1, 2, 3\}$ and so $A \subseteq B \cup C$. However $A \not\subseteq B$ because $3 \notin B$ and $A \not\subseteq C$ because $2 \notin C$. □

Do your rough work here. This will not be marked.

4. [3] Let X and Y be sets. Use the Laws of Set Theory to prove that $X^c \cup ((X \cap Y^c) \cup Y) = \mathcal{U}$.

$X^c \cup ((X \cap Y^c) \cup Y) = X^c \cup ((X \cup Y) \cap (Y^c \cup Y))$ Distributivity

$= X^c \cup ((X \cup Y) \cap \mathcal{U})$, Known Set Equality

$= X^c \cup (X \cup Y)$, Known set Equality (Identity)

$= (X^c \cup X) \cup Y$, Associativity

$= \mathcal{U} \cup Y$, Known Set Equality

$= \mathcal{U}$ Known Set Equality (Dominance)

5. [3] Prove that $(A \setminus B) \cap C^c = A \setminus (B \cup C)$ by using set builder notation and showing that the LHS and RHS are defined by logically equivalent statements. Every time you use a logical equivalence, you should state the logical equivalence that you are using.

$(A \setminus B) \cap C^c = \{x : (x \in A \setminus B) \wedge (x \in C^c)\}$

$= \{x : ((x \in A) \wedge (x \in B^c)) \wedge (x \in C^c)\}$

$= \{x : (x \in A) \wedge ((x \in B^c) \wedge (x \in C^c))\}$ Assoc.

$= \{x : (x \in A) \wedge \neg(\neg(x \in B^c) \vee \neg(x \in C^c))\}$ De Morgan

$= \{x : (x \in A) \wedge \neg((x \in B) \vee (x \in C))\}$ Double Neg.

$= \{x : (x \in A) \wedge \neg(x \in B \cup C)\}$

$= A \setminus (B \cup C)$

Do your rough work here. This will not be marked.

Name: _____ **Student ID Number:** _____

Instructions:

Please completely fill in the rectangle associated with your response. Example: ■ B C D E

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