

202401 Math 122 A01 Quiz #3

#V _____

Name: _____

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a Sharp calculator with model number beginning EL510R. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Use the blank to indicate whether each statement is true (T) or false (F). No reasons are necessary.

___ If $3, 8 \in X$ and $x + y \in X$ whenever $x, y \in X$, then $13 \in X$.

___ If $a_1 = 0$, $a_2 = 1$ and $a_n = a_{n-2} + 1$ for $n \geq 3$, then $a_1, a_2, \dots = 0, 1, 0, 2, 0, 3, 0, \dots$

___ If $S(n)$ is an open statement in the universe of positive integers and the truth of $S(k)$ logically implies the truth of $S(k + 1)$ for all $k \geq 1$, then $S(n)$ is true for all positive integers n .

___ $1^3 + 2^3 + \dots + n^3 = n^2(n + 1)^2/4$ for all $n \geq 0$.

2. [4] Let $a_0 = 1$, $a_1 = 2$, and $a_n = 4a_{n-1} - 4a_{n-2}$, $n \geq 2$. Use induction to prove that $a_n = 2^n$ for all integers $n \geq 0$.

3. [2] Give a recursive definition of the sequence $s_0, s_1, s_2, \dots = -2, 4, 10, 16, \dots, 6n-2, \dots$

4. [2] Gary is proving by induction the (true) statement that *every integer $n \geq 18$ can be written as a sum of 4s and 7s*. He first verifies the truth of the statement for $n = 18, 19, 20$ and 21 . Now he needs to state the induction hypothesis. Use the space below to help him out by telling him what to write.

5. Let $s_0 = 7$, and $s_n = 2s_{n-1} + 7$ for $n \geq 1$.

(a) [1] Use the recursive definition to find expressions for s_1, s_2, s_3, s_4 as a sum of powers multiplied by constants.

(b) [2] Use your work in part (a) to guess a closed-form formula (i.e. not just a summation) for s_n that is valid for all $n \geq 0$. You do not need to prove that your formula is correct.

6. [2] Use the blank to indicate whether each statement is true (T) or false (F). No reasons are necessary.

___ If $x \in \mathbb{R} \setminus \mathbb{Z}$ then $\lfloor x \rfloor < \lceil x \rceil$.

___ If $x \in \mathbb{R}$, then $\lfloor 2x \rfloor < 2\lfloor x \rfloor$.

___ When -41 is divided by 5 , the remainder is -1 .

___ The remainder when $(2503)_9$ is divided by 9 equals 2 .