202201 Math 122 A03 Quiz #3

#V00:________________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but are neither needed nor helpful. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Let n be an integer. Prove that *if* $3n$ *is even, then* n *is even.* Hint: the contrapositive.

   We will prove the contrapositive: if n is odd then $3n$ is odd.

   Suppose n is odd. Then $n = 2k+1$ for some integer k.

   $\therefore$ $3n = 3(2k+1) = 6k+3 = 2(3k+1)+1$

   Since $3k+1$ is an integer, $3n$ is odd.

2. Let A, B and C be sets.

   (a) [2] Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

   Take any $x \in A$.
   Since $A \subseteq B$, $x \in B$
   and since $B \subseteq C$, $x \in C$.
   $\therefore$ $A \subseteq C$

   (b) [2] If in part (a) we have $B \subsetneq C$, is it true that $A \subsetneq C$? Explain.

   Yes. If $B \subsetneq C$ then there is an element $x \in C$ such that $x \notin B$. Since $A \subseteq B$, $x \notin A$.
   $\therefore$ $A \subsetneq C$.

3. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   T $\emptyset \subseteq \{a, \{b\}, \{a, b, c\}\}$.

   F $\{a, b\} \in \{\{a\}, \{b\}, \{a, b, c\}\}$.

   T $\{x \in \mathbb{R} : x^2 + 5 = 0\} = \{n \in \mathbb{Z} : n^2 - 1 = 9\}$.

   F $\mathcal{P}(\emptyset) = \emptyset$.

4. [3] Let A and B be sets. Prove that $(A \cup B)^c = A^c \cap B^c$ using either an argument involving set builder notation, or by showing LHS $\subseteq$ RHS and RHS $\subseteq$ LHS.

$(A \cup B)^c = \{x : x \in (A \cup B)^c\}$

$= \{x : x \notin A \cup B\}$ Def'n

$= \{x : \neg(x \in A \vee x \in B)\}$ "

$= \{x : \neg(x \in A) \wedge \neg(x \in B)\}$ De Morgan

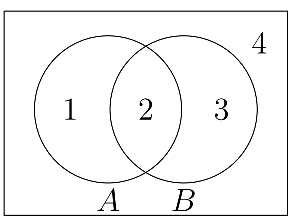
$= \{x : x \notin A \wedge x \notin B\}$ Def'n

$= \{x : x \in A^c \wedge x \in B^c\}$ "

$= \{x : x \in A^c \cap B^c\}$ "

$= A^c \cap B^c$

5. [2] Give a counterexample to show that $(B^c \setminus A)^c \neq B \setminus A^c$ for all sets A and B.



Let $\mathcal{U} = \{1,2,3,4\}$, $A = \{1,2\}$, $B = \{2,3\}$

Then $(B^c \setminus A)^c = (\{1,4\} \setminus \{1,2\})^c$

$= \{4\}^c = \{1,2,3\}$

and $B \setminus A^c = \{2,3\} \setminus \{3,4\} = \{2\}$

$\therefore (B^c \setminus A)^c \neq B \setminus A^c$ for all sets A & B

6. [2] Let A and B be sets. Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

T $A \setminus B = (A \oplus B) \cap A$.

T Every set A has exactly one subset which is not is a proper subset of A.

F If $A \subseteq B$, then $A \cup B = A$.

F If $A \oplus B \neq \emptyset$ then $A \neq \emptyset$.