201709 Math 122 A01 Quiz #3

#V00:________________ Name: Key

This quiz has 2 pages and 5 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   T If $\neg p$ logically implies $q \wedge \neg q$, then p is true.

   F When an integer is divided by 10, the remainder is its rightmost digit.

   F There is an integer $n > 1$ which has no prime divisor.

   F If $\frac{a}{b}$ is an integer, then $a \mid b$.

2. [3] Let a and b be integers, where a is odd. Prove that if ab is odd, then b is odd. (Hint: the contrapositive.)

   Suppose b is even.
   Then $b = 2k$ for some integer k.
   $\therefore$ $ab = 2ka$
   Since ka is an integer, ab is even.

3. [3] Let n be an integer such that the remainder when n is divided by 4 equals 3. Is $n^2 + 5$ is a multiple of 8? Why or why not?

   We are given that $n = 4q + 3$ for some integer q.

   $\therefore$ $n^2 + 5 = (4q+3)^2 + 5$
   $= (16q^2 + 24q + 9) + 5 = 16q^2 + 24q + 14$
   $= 8(2q^2 + 3q + 1) + 6$

   Since $2q^2 + 3q + 1$ is an integer, we have by the Division Algorithm that the remainder when n^2+5 is divided by 8 equals 6
   $\therefore$ $n^2 + 5$ is not a multiple of 8.

4. [3] Let a, b, c be integers such that $a \mid b$ and $b \mid c$. Prove that $a \mid c$.

Since $a|b$, there is an integer k s.t. $b = ka$
" $b|c$ " " " " ℓ " " $c = b\ell$

$\therefore$ $c = b\ell = (ak)\ell = a(k\ell)$
Since $k\ell$ is an integer, $a|c$

5. [4] Prove by induction that $2 + 4 + \cdots + (2n) = n(n+1)$, for all integers $n \geq 1$.

<u>Basis</u>: When $n=1$ we have LHS $= 2$ &
RHS $= 1(1+1) = 2$. $\therefore$ The stmt is true when $n=1$.

<u>IH</u>: For some $k \geq 1$, assume $2+4+\cdots+2t = t(t+1)$ for $t = 1, 2, \ldots, k$.

<u>IS</u> We want to show
$$2+4+\cdots+2(k+1) = (k+1)((k+1)+1)$$

Consider the LHS:

$$2+4+\cdots+2(k+1)$$
$$= \underbrace{2+4+\cdots+2k} + 2(k+1)$$
$$= k(k+1) + 2(k+1) \quad \text{by IH}$$
$$= (k+1)(k+2), \text{ as wanted}$$

$\therefore$ By PMI, $2+4+\cdots+2n = n(n+1)$ $\forall n \geq 1$