201809 Math 122 A01 Quiz #3

#V00:________________ Name: Solutions

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   F For the universe of integers, what is the truth value of $\exists y, \forall x, xy \geq x^2$?

   F When the sentence "*Everybody loves somebody sometime.*" is written in symbols, a total of two quantifiers appear.

   F The negation of the statement "*Every true-false question is easy or false*" is "*Some true-false questions are difficult and false.*"

   T For the universe $\mathcal{U} = \{1, 2, 3\}$, the statement $\neg\exists x, 5x < 10$ is logically equivalent to $(5 \cdot 1 \geq 10) \wedge (5 \cdot 2 \geq 10) \wedge (5 \cdot 3 \geq 10)$.

2. [3] Let n be an integer. Prove that if $5n$ is odd, then n is odd.
   (Hint: the contrapositive.)

   We prove the contrapositive: if n is even then $5n$ is even.

   Suppose n is even.
   Then $n = 2k$ for some integer k.
   $\therefore 5n = 5(2k) = 2 \cdot (5k)$
   Since $5k$ is an integer, $5n$ is even

   $\therefore$ if $5n$ is odd, then n is odd.

3. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary. Let $X = \{1, \{3\}, \{2, \{3, 4\}\}\}$.

   T $\{3\} \in X$

   F $\{2, \{3, 4\}\} \subseteq X$.

   T The power set of X, $\mathcal{P}(X)$, has exactly 8 elements.

   T $\emptyset \subsetneq X$.

4. [3] Let A, B, and C be sets. Prove that $(A \cup B) \cup C = A \cup (B \cup C)$ by using set builder notation and showing that the LHS and RHS are defined by logically equivalent expressions.

$$
\begin{aligned}
(A \cup B) \cup C &= \{x : (x \in A \cup B) \vee (x \in C)\} \\
&= \{x : ((x \in A) \vee (x \in B)) \vee (x \in C)\} \\
&= \{x : (x \in A) \vee ((x \in B) \vee (x \in C))\} \\
&= \{x : (x \in A) \vee (x \in B \cup C)\} \\
&= A \cup (B \cup C)
\end{aligned}
$$

5. [3] Let A and B be sets. Prove that if $A \subseteq B$, then $B^c \subseteq A^c$.

Suppose $A \subseteq B$.

Take any $x \in B^c$.

Then $x \notin B$. Since $A \subseteq B$, $x \notin A$.

$\therefore$ $x \in A^c$

$\therefore$ $B^c \subseteq A^c$

6. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary. Let A, B, C be sets.

T $A \cap B \subseteq A$.

T $B \subseteq A \cup B$.

T If $A \setminus B = \emptyset$, then $A \subseteq B$.

T If $A \oplus B \neq \emptyset$, then $A \neq B$.