201909 Math 122 A01 Quiz #3

#V00:______________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   F $\emptyset \in \{a, \{b\}, \{a, b, c\}\}$.

   F $\{a, b\} \subseteq \{a, \{b\}, \{a, b, c\}\}$.

   T $\{x \in \mathbb{R} : x^2 + 1 = 0\} = \{n \in \mathbb{Z} : n^2 - 1 = 7\}$.

   T $\mathcal{P}(\emptyset) \neq \emptyset$.

2. Let A, B and C be sets.

   (a) [2] Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

   Take any $x \in A$.
   Since $A \subseteq B$, $x \in B$.
   Since $B \subseteq C$, $x \in C$.
   $\therefore A \subseteq C$

   (b) [2] If in part (a) we have $A \subsetneq B$, is it true that $A \subsetneq C$? Explain.

   Yes. If $A \subsetneq B$ then there exists $y \in B$ s.t. $y \notin A$. Since $B \subseteq C$, $y \in C$.
   $\therefore A \neq C$.
   Since $A \subseteq C$, this means $A \subsetneq C$.

3. [2] Answer **True** or **False** and briefly explain your reasoning. *If* $A \oplus B \neq \emptyset$ *then* $A \neq B$.

   True. $A \oplus B = A \setminus B \cup B \setminus A$.
   If this set is not empty then $A \setminus B \neq \emptyset$ or $B \setminus A \neq \emptyset$. $\therefore$ One of A, B has an element not in the other, so they are not equal.

4. [3] Let A and B be sets. Use any method to prove that $(A \cap B)^c = A^c \cup B^c$. (Note. A Venn Diagram is not acceptable as a proof.)

(LHS $\subseteq$ RHS) Take any $x \in (A \cap B)^c$. Then $x \notin A \cap B$. $\therefore x \notin A$ or $x \notin B$.

$\therefore x \in A^c$ or $x \in B^c$

$\therefore x \in A^c \cup B^c$, so $(A \cap B)^c \subseteq A^c \cup B^c$

(RHS $\subseteq$ LHS) Take any $x \in A^c \cup B^c$.
Then $x \in A^c$ or $x \in B^c$.

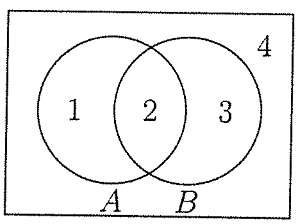
$\therefore x \notin A$ or $x \notin B$

$\therefore x \notin A \cap B$

$\therefore x \in (A \cap B)^c$, so $A^c \cup B^c \subseteq (A \cap B)^c$

$\therefore (A \cap B)^c = A^c \cup B^c$

5. [2] Use the Venn diagram below to investigate whether $(B^c \setminus A)^c$ equals $B \setminus A^c$. If the statement is true, explain your reasoning. If the statement is false, then give a counterexample that demonstrates it is false.



$(B^c \setminus A)^c$: reg 1, 2, 3
$B \setminus A^c$: reg 2
$\therefore$ Think it is false

Counterexample $\mathcal{U} = \{1, 2, 3, 4\}$
$A = \{1, 2\}$, $B = \{2, 3\}$. Then
$(B^c \setminus A)^c = \{1, 2, 3\} \neq \{2\} = B \setminus A^c$.

6. [2] Let A and B be sets. Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

T $A \setminus B = (A \oplus B) \setminus B$.

T A set with n elements has exactly $2^n - 1$ proper subsets.

T If $A \subseteq B$, then $A \cap B = A$.

T If $A \oplus B = B$ then $A = \emptyset$.