201609 Math 122 [A03] Quiz #3

#V00:________________ Name: Key

This quiz has 2 pages and 5 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Let $A = \{1, 2, \{3, 4\}\}$ and $B = \{2, 3, \{1, 4\}\}$ Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   F $3 \in A$.

   F $\{1, 4\} \subseteq B$.

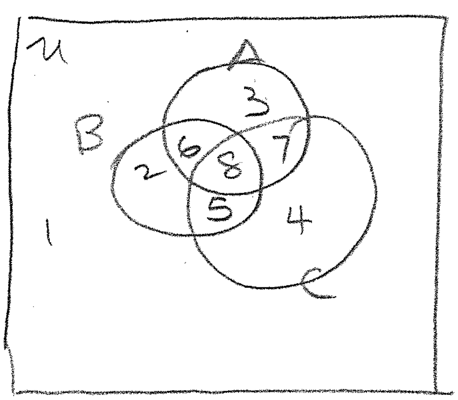
   T $\emptyset \subsetneq A$.

   T $\{2, 3\} \in \mathcal{P}(B)$.

2. [4] Prove that, for all sets A and B, $(A \setminus B) \cup (A \cap B) = A$.

$$
\begin{aligned}
(A \setminus B) &\cup (A \cap B) \\
&= (A \cap B^c) \cup (A \cap B) && \text{Known} \\
&= A \cap \underbrace{(B^c \cup B)}_{\mathcal{U}} && \text{Dist've} \\
&= A && \text{Identity}
\end{aligned}
$$

3. [3] Give a counterexample to the statement $(A \setminus B) \cap C = (A \cap C) \setminus B^c$, for all sets A, B and C.



Let: $\mathcal{U} = \{1, 2, \ldots, 8\}$

$A = \{3, 6, 7, 8\}$

$B = \{2, 5, 6, 8\}$

$C = \{4, 5, 7, 8\}$

Then $(A \setminus B) \cap C = \{7\}$

and $(A \cap C) \setminus B^c = \{8\}$

$\therefore$ The sets are not equal in general.

4. [4] Let A, B and C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(LHS $\subseteq$ RHS) Take any $(x,y) \in A \times (B \cap C)$.
Then $x \in A$ and $y \in B \cap C$
$\therefore$ $y \in B$ and $y \in C$

$\therefore$ $(x,y) \in A \times B$ and $(x,y) \in A \times C$
$\therefore$ $(x,y) \in (A \times B) \cap (A \times C)$
$\therefore$ $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$

(RHS $\subseteq$ LHS) Take any $(x,y) \in (A \times B) \cap (A \times C)$
Then $(x,y) \in A \times B$ and $(x,y) \in A \times C$
$\therefore$ $x \in A$ and $y \in B$, & $x \in A$ and $y \in C$
$\therefore$ $y \in B \cap C$
$\therefore$ $(x,y) \in A \times (B \cap C)$
$\therefore$ $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$

$\therefore$ $A \times (B \cap C) = (A \times B) \cap (A \times C)$

5. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

T $B \subseteq A \cup B$ for all sets A and B.
T If A and B are sets and $A \subsetneq B$, then $A \oplus B \neq \emptyset$.
F If $A = \{1, 2\}$ and $B = \{a, b\}$, then $\{1, b\} \in A \times B$.
T If A and B are sets and $A \times B \neq \emptyset$, then $A \neq \emptyset$ and $B \neq \emptyset$.