201801 Math 122 A01 Quiz #3

#V00:________________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   T When the statement "*Every real number has a negative*" is written in symbols, both a universal and existential quantifier are used.

   T For the universe of integers, when the statement "*if m is even and n is even, then mn is even*" is written in symbols, a total of five quantifiers appear.

   F The negation of the statement "*Some triangles have three equal angles*" is "*Some triangles have no two equal angles.*"

   F $\neg\forall x, \forall y, s(x) \wedge s(y)$ is logically equivalent to $\exists x, \forall y, \neg s(x) \vee \neg s(y)$.

2. [2] Suppose that m and n are integers. It is claimed that the argument below proves that *if mn is odd, then m and n are both odd.* Does it? Explain your reasoning.

   Suppose that the integers m and n are both even. Then there exists an integer k such that $m = 2k$, and there exists an integer ℓ such that $n = 2\ell$. Thus,

   $$mn = (2k)(2\ell) = 2(2k\ell).$$

   Since $2k\ell$ is an integer, mn is even.

   No. The statement which begins the argument is not the contrapositive of the statement to prove; "and" should be "or".

3. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   F $2 \in \{1, \{2\}, \{1, 2\}\}$.

   T If $A \cap B = \emptyset$, then $A \setminus B = A$.

   F $\{3, 4\} \in \mathcal{P}(\{1, 2, \{3, 4\}\})$.

   T $\emptyset$ has no proper subsets.

4. [2] Let A and B be sets. Prove that $(A \cap B)^c = A^c \cup B^c$ by using set builder notation and showing that the LHS and RHS are defined by logically equivalent expressions.

$$
\begin{aligned}
(A\cap B)^c &= \{x : x \notin A \cap B\} \\
&= \{x : \neg(x \in A \cap B)\} \\
&= \{x : \neg(x \in A \wedge x \in B)\} \\
&= \{x : \neg(x \in A) \vee \neg(x \in B)\} \\
&= \{x : x \notin A \vee x \notin B\} \\
&= \{x : x \in A^c \vee x \in B^c\} = A^c \cup B^c
\end{aligned}
$$

5. [3] Let X and Y be sets. Use the Laws of Set Theory to prove that $(X \cup Y) \setminus Y = X \setminus Y$.

$$
\begin{aligned}
(X \cup Y)\setminus Y &= (X \cup Y) \cap Y^c && \text{Known} \\
&= (X \cap Y^c) \cup \cancel{(Y \cap Y^c)}^{\emptyset} && \text{Dist} \\
&= X \cap Y^c && \text{Known, identity} \\
&= X \setminus Y
\end{aligned}
$$

6. [4] Let A and B be sets such that $A \cup B = B$. Prove that $A \subseteq A \cap B$. Are the sets A and $A \cap B$ actually equal? Why or why not?

Take any $x \in A$.
Then $x \in A \cup B$ by def'n of union
Since $A \cup B = B$, $x \in B$
$\therefore x \in A \cap B$
$\therefore A \subseteq A \cap B$.

Yes, the sets are equal.
It is always true that $A \cap B \subseteq A$
(by def'n of intersection).
$\therefore A = A \cap B$ (when $A \cup B = B$)