

202409 Quiz 4
Math 122 A04
Instructor: Jonathan Noel

Do not open the booklet before you are told to

Date and Time: Tuesday, October 29, 2024 at 1:55pm.

Instructions: There are 3 pages and 4 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the sheet for rough work.

True/False Instructions: Question 1 consists of 8 true/false questions labelled **TF 1** to **TF 8**. The last page of your test booklet is a bubble sheet for answering them. You may detach the back page from the rest of the test if you wish. Only fill in a bubble for questions 1-8 on the bubble sheet. When making your selection, **True is A** and **False is B**. Do not select C, D or E. If you want to change your answer after filling in a bubble, then please erase your previous answer or write something on the sheet to try to make your final selection as clear as possible.

Do your rough work here. This will not be marked.

1. [4] Use the bubble sheet provided on the last page of the test booklet to indicate whether each statement is **True (A)** or **False (B)**.

Let $A = \{1, 2, \dots, 8\}$.

[TF 1] The number of proper subsets of A containing 2 is $2^7 - 1$.

[TF 2] The number of non-empty subsets of A containing 2 is $2^7 - 1$.

[TF 3] $\{5\}$ is a subset of the power set of A .

[TF 4] If $B \subseteq A$, then $|A \setminus B| < 8$.

[TF 5] $4^0 + 4^1 + \dots + 4^n = 5^n$ for all integers $n \geq 0$.

[TF 6] If $x, y \in \mathbb{Q}$ such that $\lfloor x \rfloor < \lceil y \rceil$, then $x < y$.

[TF 7] Let $a_0, a_1, \dots$ be the sequence recursively defined by $a_0 = 0$, $a_1 = 3$ and $a_n = 3a_{n-1} - a_{n-2}$ for all $n \geq 2$. Then $a_3 = 27$.

[TF 8] The base 8 representation of 3000 is $(5607)_8$.

2. [1] Give a recursive definition of the sequence $a_1, a_2, \dots$, where $a_n = 1^1 + 2^2 + 3^3 + \dots + n^n$, for every $n \geq 1$.

Do your rough work here. This will not be marked.

3. [5] Let $a_0, a_1, \dots$ be the sequence recursively defined by $a_0 = 0$, $a_1 = 2$ and $a_n = 2a_{n-1} - a_{n-2}$ for all $n \geq 2$. Prove by induction that $a_n = 2n$ for all integers $n \geq 0$.

4. [5] Use induction to prove that $1 + 4 + 7 + \dots + (3n - 2) = \frac{n(3n-1)}{2}$ for all integers $n \geq 1$.

Do your rough work here. This will not be marked.