202401 Math 122 A01 Quiz #4

#V________________ Name: Key

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a Sharp calculator with model number beginning EL510R. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

   F The exponent of 3 in the prime factorization of 12! equals 4.

   F There are integers x and y such that $12x + 15y = 13$.

   F Any positive integer $n > 1$ has a prime divisor less than or equal to $\sqrt{n}$.

   F If $a = 2^4 \cdot 5^3 \cdot 7$ and $b = 2^2 \cdot 5^6 \cdot 11$, then $lcm(a, b) = 2^2 \cdot 5^3 \cdot 7 \cdot 11$.

2. [3] Let $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

   Suppose $a|b$ and $b|c$.
   Then there are integers k and l such that $ak=b$ and $bl=c$.
   $\therefore$ $c = bl = (ak)l = a(kl)$
   Since $k, l \in \mathbb{Z}$, $kl \in \mathbb{Z}$.
   $\therefore$ $a|c$

3. [1] Suppose $n > 1$ has the prime factorization $p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$. What is the prime factorization of n^3?

   $p_1^{3e_1} p_2^{3e_2} \cdots p_k^{3e_k}$

4. [2] Suppose $n = (2 \cdot 3 \cdot 11 \cdot 17)^{122}$. Use the Fundamental Theorem of Arithmetic to explain why there is no integer k such that $21k = n$.

   The prime factorization of n contains no 7.
   The prime factorization of $21k$ contains 7.
   If there were an integer k so that $21k=n$ then n would have 2 different prime factorizations, contrary to the FTA.

5. [3] Use the Euclidean Algorithm to find $d = gcd(122, 580)$, and then use your work to find integers x and y such that $122x + 580y = d$.

$580 = 4 \cdot 122 + 92$

$122 = 1 \cdot 92 + 30$

$92 = 3 \cdot 30 + 2 \leftarrow \therefore gcd(122, 580) = 2$

$30 = 15 \cdot 2 + 0$

$\therefore$ We have

$2 = 92 - 3 \cdot 30 = 92 - 3 \cdot (122 - 1 \cdot 92)$

$= 4 \cdot 92 - 3 \cdot 122$

$= 4(580 - 4 \cdot 122) - 3 \cdot 122$

$= 4 \cdot 580 - 19 \cdot 122$

$= 580 \cdot \underbrace{4}_{x} + 122 \cdot \underbrace{(-19)}_{y}$

6. [2] Use any method to prove that $gcd(a, a + 1) = 1$ for any integer $a > 1$.

$(a+1) \cdot 1 + a(-1) = 1$

Since there exist integers x & y s.t. $(a+1)x + ay = 1$, $gcd(a, a+1) = 1$

7. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

F For any integers d, a, b, if $d \mid ab$ then $d \mid a$ or $d \mid b$.

T None of the odd primes p_1, p_2, p_3 divide $p_1 p_2 p_3 + 2$.

F The last digit of 99^{122} is 9.

F $4 \times 25 + 6 \times 15^5 - 8 \equiv 8 \pmod 5$.