

202401 Math 122 A01 Quiz #4

#V_____

Name: _____

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a Sharp calculator with model number beginning EL510R. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

- ___ The exponent of 3 in the prime factorization of $12!$ equals 4.
- ___ There are integers x and y such that $12x + 15y = 13$.
- ___ Any positive integer $n > 1$ has a prime divisor less than or equal to $\sqrt{n}$.
- ___ If $a = 2^4 \cdot 5^3 \cdot 7$ and $b = 2^2 \cdot 5^6 \cdot 11$, then $\text{lcm}(a, b) = 2^2 \cdot 5^3 \cdot 7 \cdot 11$.

2. [3] Let $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

3. [1] Suppose $n > 1$ has the prime factorization $p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$. What is the prime factorization of n^3 ?

4. [2] Suppose $n = (2 \cdot 3 \cdot 11 \cdot 17)^{122}$. Use the Fundamental Theorem of Arithmetic to explain why there is no integer k such that $21k = n$.

5. [3] Use the Euclidean Algorithm to find $d = \gcd(122, 580)$, and then use your work to find integers x and y such that $122x + 580y = d$.

6. [2] Use any method to prove that $\gcd(a, a + 1) = 1$ for any integer $a > 1$.

7. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

- ___ For any integers d, a, b , if $d \mid ab$ then $d \mid a$ or $d \mid b$.
- ___ None of the odd primes p_1, p_2, p_3 divide $p_1 p_2 p_3 + 2$.
- ___ The last digit of 99^{122} is 9.
- ___ $4 \times 25 + 6 \times 15^5 - 8 \equiv 8 \pmod{5}$.