202201 Math 122 A03 Quiz #4

#V00:_________________ Name: Key (Joseph)

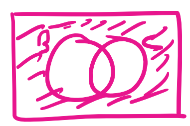
This quiz has 2 pages and 5 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [3] Let A and B be sets such that $A \cup B = A \cap B$. Prove that $A \subseteq B$ using an argument that starts with "*Take any* $x \in A$ ...". Are the sets A and B equal?

   Take any $x \in A$. Because $A \subseteq A \cup B$ (by definition), $x \in A \cup B$. Since $A \cup B = A \cap B$, $x \in A \cap B$, which means that $x \in B$ also. Thus $A \subseteq B$. ■

   Yes, $A = B$; the above argument could be run with A and B swapped to show that $B \subseteq A$.

2. [3] A math professor goes for coffee 122 times during the term. He goes 60 times with Dr. B, 50 times with Dr. C, 23 times with both of them, and alone the remaining times. How many times does he go for coffee alone?

   If B = the set of times he goes w/ Dr. B,
   C = - - - - - - - - - - Dr. C,
   then we want $|B^c \cap C^c|$; by inclusion-exclusion we have

   $$|B^c \cap C^c| = 122 - |B \cup C| = 122 - (60 + 50 - 23) = \boxed{35}.$$

3. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   T The number of non-empty subsets of $\{a, b, c, d, e\}$ that contain a equals 2^4. irrelevant, as a is in each subset

   F The number of proper subsets of $\{a, b, c, d, e\}$ that contain a and do not contain b equals 7. already proper (it's 8)

   F If $S(n)$ is true for $n = 1, 2, 3$, and $S(1) \wedge S(2) \wedge S(3) \Rightarrow S(4)$, then the statement $S(n)$ is true for all integers $n \geq 1$. Need more than just the first implication.

   F If $S(1) \wedge S(2) \wedge \cdots \wedge S(k) \Rightarrow S(k+1)$ for all integers $k \geq 1$, then the statement $S(n)$ is true for all integers $n \geq 1$. Need base cases!

4. Give a recursive definition of the sequence $s_{\cancel{0}\,1}, s_{\cancel{1}\,2}, s_{\cancel{2}\,3}, \ldots$

(a) [1] where the sequence is $1, 5, 9, 13, \ldots$.

Let $s_{\cancel{0}\,1} = 1$ and $s_n = s_{n-1} + 4$ for $n \geq \cancel{1}\,2$

(b) [1] where $s_n = 1 + 2 + \cdots + n$.

Let $s_{\cancel{0}\,1} = \cancel{0}\,1$ and $s_n = s_{n-1} + n$ for $n \geq \cancel{1}\,2$

5. [5] Use induction to prove that every positive integer $n \geq 6$ can be written as a sum of threes and fours, - - - -

Basis: Note that $6 = 3+3$, $7 = 3+4$, and $8 = 4+4$, so the integers 6, 7, 8 are all sums of 3's and 4's.

Induction Hypothesis: Suppose that for some $k \geq 8$, every integer between 6 and k is a sum of 3's and 4's.

Induction Step: Consider the integer $k+1$. Rewrite $k+1 = (k-2)+3$. Since $k \geq 8$, we know $k-2 \geq 6$, so by our Ind. Hyp., $k-2$ is a sum of 3's and 4's. But then $k+1 = (k-2)+3$ is also a sum of 3's and 4's, as desired.

Conclusion: By induction, every integer $n \geq 6$ can be written as a sum of 3's and 4's. $\blacksquare$