201701 Math 122 [A01] Quiz #4

#V00:________________ Name: Key

This quiz has 2 pages and 5 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   T No relation on $\{1,2,3,4,5\}$ that contains exactly 4 ordered pairs can be reflexive.

   T If $\mathcal{R}$ is an anti-symmetric relation on a set A, and there exist $x, y \in A$ such that $(x,y) \in \mathcal{R}$ and $x \neq y$, then $(y,x) \notin \mathcal{R}$.

   T If $\mathcal{R}$ is a symmetric relation on a set A, and there exist $x, y \in A$ such that $(x,y) \in \mathcal{R}$ and $x \neq y$, then $(y,x) \in \mathcal{R}$.

   T For any set A there is a relation on A that has all four of the properties reflexive, anti-symmetric, symmetric, transitive.

2. Let $A = \{1,2,3\}$. Let $\mathcal{R}$ be a reflexive, antisymmetric and transitve relation on A. Suppose $(2,1),(3,2) \in \mathcal{R}$.

   (a) [1] Explain why $(1,2)$ and $(2,3)$ are not in $\mathcal{R}$.

   $\mathcal{R}$ is antisymmetric and $(2,1) \in \mathcal{R}$. Since $1 \neq 2$, $(1,2) \notin \mathcal{R}$. Similarly for $(2,3)$

   (b) [1] Is $(1,3)$ in $\mathcal{R}$? Why or why not?

   No. $(3,2),(2,1) \in \mathcal{R}$ $\therefore$ $(3,1) \in \mathcal{R}$ by transitivity $\therefore$ $(1,3) \notin \mathcal{R}$ for the same reasons as in (a)

   (c) [1] Write $\mathcal{R}$ as a set of ordered pairs.

   $\mathcal{R} = \{(1,1),(2,2),(3,3),(2,1),(3,2),(3,1)\}$

3. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   F Any function $f : \{1,2,3,4\} \to \{1,2,3,4,5\}$ contains exactly 5 ordered pairs.

   F Any relation on $\{1,2,3,4\}$ that contains exactly 4 ordered pairs is a function.

   T A function $f : A \to B$ is onto if and only if $\text{rng} f = B$.

   T There is a function $f : \{1,2,3,4\} \to \{1,2,3,4\}$ which is a symmetric relation on $\{1,2,3,4\}$.

4. [4] Let $\mathcal{F}$ be the relation on $\mathbb{N}$ such that $(x, y) \in \mathcal{F}$ if and only if $\lfloor \frac{x}{4} \rfloor = \lfloor \frac{y}{4} \rfloor$. Prove that $\mathcal{F}$ is an equivalence relation and find the equivalence class $[10]$.

Let $x \in \mathbb{N}$. Since $\lfloor \frac{x}{4} \rfloor = \lfloor \frac{x}{4} \rfloor$ we have $x \mathcal{F} x$.
$\therefore$ $\mathcal{F}$ is reflexive

Suppose $x \mathcal{F} y$. Then $\lfloor \frac{x}{4} \rfloor = \lfloor \frac{y}{4} \rfloor$. $\therefore \lfloor \frac{y}{4} \rfloor = \lfloor \frac{x}{4} \rfloor$,
so $y \mathcal{F} x$. $\therefore$ $\mathcal{F}$ is symmetric.

Suppose $x \mathcal{F} y$ and $y \mathcal{F} z$. Then $\lfloor \frac{x}{4} \rfloor = \lfloor \frac{y}{4} \rfloor$
and $\lfloor \frac{y}{4} \rfloor = \lfloor \frac{z}{4} \rfloor$. $\therefore \lfloor \frac{x}{4} \rfloor = \lfloor \frac{z}{4} \rfloor$, so $x \mathcal{F} z$.
$\therefore$ $\mathcal{F}$ is transitive

$\therefore$ $\mathcal{F}$ is an equivalence relation

$[10] = \{ y : \lfloor \frac{y}{4} \rfloor = \lfloor \frac{10}{4} \rfloor \} = \{8, 9, 10, 11\}$

5. Let $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(x) = 3x + 2$.

(a) [2] Prove that f is 1-1.

Suppose $f(x_1) = f(x_2)$

$$
\begin{aligned}
\text{Then} \quad 3x_1 + \cancel{2} &= 3x_2 + \cancel{2} \\
\therefore \quad 3x_1 &= 3x_2 \\
\therefore \quad x_1 &= x_2
\end{aligned}
$$

$\therefore$ f is 1-1

(b) [2] Give a reason that explains why f is not onto.

$1 \notin \operatorname{rng} f$. We have $f(x) = 1$
$\iff 3x + 2 = 1 \iff x = -\frac{1}{3}$

Since $-\frac{1}{3} \notin \mathbb{N}$, f is not onto.