201609 Math 122 [A03] Quiz #4

#V00:________________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   T If there are 2^9 relations on A, then the set A has exactly 3 elements.

   T Exactly one function $f : \mathbb{R} \to \mathbb{R}$ is a reflexive relation on $\mathbb{R}$.

   F No relation on $A = \{1, 2, 3\}$ is both symmetric and antisymmetric.

   T If $f : \{1, 2, 3\} \to \{1, 2, 3\}$ is a function and $(1, 2), (2, 3) \in f$, then f is not a transitive relation on $\{1, 2, 3\}$.

2. Let $\mathcal{I} = \{(x, y) : x, y \in \mathbb{R} \text{ and } x < y\}$, so that $\mathcal{I}$ is the collection of non-empty open intervals of real numbers. Let $\mathcal{R}$ be the relation on $\mathcal{I}$ where

$$(x, y)\,\mathcal{R}\,(v, w) \Leftrightarrow x \le v \text{ and } y \ge w.$$

   (a) [1] Show that $\mathcal{R}$ is not symmetric.

   $(0,3)\,\mathcal{R}\,(1,2)$ b/c $0 \le 1$ and $3 \ge 2$

   but $(1,2) \not\mathcal{R} (0,3)$ as $1 \not\le 0$.

   $\therefore \mathcal{R}$ is not symmetric.

   (b) [3] Prove that $\mathcal{R}$ is anti-symmetric.

   Suppose $(x,y)\,\mathcal{R}\,(u,v)$ & $(u,v)\,\mathcal{R}\,(x,y)$.

   Then $x \le u$ and $y \ge v$

   and $u \le x$ and $v \ge y$

   $\therefore u = x$ and $v = y$

   $\therefore (x,y) = (u,v)$

   $\therefore \mathcal{R}$ is anti-symmetric.

3. [2] Let $\sim$ be the relation on $\{10, 11, \ldots, 99\}$ where $x \sim y$ if and only of the sum of the digits of x equals the sum of the digits of y. (For example $16 \sim 34$ because $1+6 = 3+4$.) Take it as given that $\sim$ is an equivalence relation. How many different equivalence classes does it have? Justify your answer.

18: one for each of the possible sums; all of them arise: 10, 11, ..., 19, 92, 93, ..., 99.

4. Let $X = \{1, 2, 3, 4, 5\}$. Let $f : X \to X$ be defined by $f(x) = \lfloor \frac{x}{2} \rfloor + \lceil \frac{x}{3} \rceil$.

   (a) [1] Write f as a set of ordered pairs.

   $f = \{(1,1), (2,2), (3,2), (4,4), (5,4)\}$

   (b) [1] Is f onto? Justify your answer.

   No. For example $3 \notin \text{rng } f$.

5. [3] Let $f : [0, \infty) \to [2, \infty)$ be defined by $f(x) = x^2 + 2$. Prove that f is 1-1.

Suppose $f(x_1) = f(x_2)$ for $x_1, x_2 \geq 0$.

Then $2 + x_1^2 = 2 + x_2^2$

so $x_1^2 = x_2^2$

and $x_1 = \pm |x_2|$

But $x_1, x_2 \geq 0$ so $x_1 = x_2$.

$\therefore f$ is 1-1.

6. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

F If $f : A \to \{0, 1, 2\}$ has more than 3 ordered pairs, then f is onto.

F No function $f : \mathbb{Z} \to \mathbb{N}$ is 1-1.

T If $f : \{a_1, a_2, \ldots, a_m\} \to \{b_1, b_2, \ldots, b_n\}$ is bijective, then $m = n$.

F If $f : \{1, 2, \ldots 10\} \to \{1, 2, \ldots, 10\}$ is a function, then so is $g = \{(b, a) : f(a) = b\}$.