

## 201809 Math 122 A01 Quiz #5

#V00: \_\_\_\_\_

Name: \_\_\_\_\_

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

\_\_\_ For  $x \in \mathbb{R}$ ,  $\lfloor x/2 \rfloor = \lceil x/2 \rceil$  if and only if  $x$  is an even integer.

\_\_\_ When  $-102$  is divided by  $-10$ , the remainder is  $-2$ .

\_\_\_ The exponent of 2 in the prime factorization of  $(8!) \cdot (9!)$  is 14.

\_\_\_  $(1001110)_2 = (96)_{16}$ .

2. [2] Find the base 9 representation of 2018.

3. [3] Let  $a, b, d \in \mathbb{Z}$ . Prove that if  $d \mid a$  and  $d \mid b$ , then  $d^2 \mid ab$ .

4. [2] Let  $n = 2^{100}3^{200}$ . Explain why the Fundamental Theorem of Arithmetic implies that there is no integer  $k$  such that  $15k = n$ .

5. (a) [3] Use the Euclidean Algorithm to find  $d = \gcd(824, 122)$ , and then use your work to find integers  $x$  and  $y$  such that  $122x + 824y = d$ .

(b) [1] Use your answer from (a) to find  $\text{lcm}(824, 122)$ .

6. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

\_\_\_ There is an integer  $b$  such that  $(23)_b = (32)_4$ .

\_\_\_ If  $a, b \in \mathbb{Z}$  and  $\gcd(a, b) = 4$ , then there are integers  $x, y$  such that  $ax + by = 12$ .

\_\_\_  $0 \mid 0$ .

\_\_\_ If  $a = 2^4 \cdot 5^3 \cdot 7$  and  $b = 2^2 \cdot 5^6 \cdot 11$ , then  $\gcd(a, b) = 2^2 \cdot 5^3$ .