201909 Math 122 A01 Quiz #5

#V00:_____ Name: _____

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, and may even be useful. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

   ___ For all $n \in \mathbb{Z}$, $2\lfloor n/2 \rfloor = n$ if and only if n is even.

   ___ $(1011001)_2 = (B1)_{16}$.

   ___ The exponent of 3 in the prime factorization of 9! is 4.

   ___ $0 \mid 3$.

2. [2] Find the base 11 representation of 1219. Use δ for the base 11 digit corresponding to ten.

3. [3] Let $a, d \in \mathbb{Z}$. Prove that if $d \mid a$, then $3d \mid 6a$.

4. (a) [2] Suppose there are integers a and b such that $3a = 2b$. Use the Fundamental Theorem of Arithmetic to explain why b must be a multiple of 3.

   (b) [1] Explain why there are no positive integers a and b such that $5^a = 7^b$.

5. (a) [2] Use the Euclidean Algorithm to find $d = gcd(1254, 228)$.

   (b) [1] Use your answer from (a) to find $lcm(1254, 228)$.

6. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

   ___ If $a, b \in \mathbb{Z}$ are relatively prime, then there exist $x, y \in \mathbb{Z}$ such that $ax + by = 2$.

   ___ $100 \equiv -8 \pmod{12}$.

   ___ $23 \cdot 122 + 3 \cdot 10^9 \equiv 9 \pmod{11}$.

   ___ If $a \in \mathbb{Z}$ and $a \equiv 5 \pmod{9}$, then the remainder when a is divided by 9 equals 4.