202401 Math 122 A01 Quiz #5

#V________________ Name: Key

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a Sharp calculator with model number beginning EL510R. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Let $\mathcal{R}$ be a relation on $A = \{1, 2, 3, 4\}$. Answer each question **TRUE** (T) or **FALSE** (F). No reasons are necessary.

   F If $1 \in A$ and $4 \in B$, then $1 \times 4 \in A \times B$.

   F If $|A| = m$ and $|B| = n$, then $|A \times B| = m^n$.

   T If $\mathcal{R}$ is symmetric and transitive, and $(3, 2) \in \mathcal{R}$, then $(2, 2) \in \mathcal{R}$.

   T If $\mathcal{R}$ is antisymmetric and transitive, and $(2, 4), (4, 3) \in \mathcal{R}$, then $(3, 2) \notin \mathcal{R}$.

2. Let $\sim$ be the relation on $A = \{10, 11, \ldots, 99\}$ defined by $x \sim y \Leftrightarrow$ the sum of the digits in the decimal representation of x equals the sum of the digits in the decimal representation of y.

   (a) [4] Prove that $\sim$ is an equivalence relation.

   Reflexive: Let $x \in A$. Since the sum of the digits of x = the sum of the digits of x, $x \sim x$. $\therefore \sim$ is reflexive.

   Symmetric: Suppose $x \sim y$. Then the sum of the digits of x = the sum of the digits of y. $\therefore$ The sum of the digits of y = the sum of the digits of x. $\therefore y \sim x$ & $\sim$ is symmetric.

   Transitive: Suppose $x \sim y$ & $y \sim z$. Then the sum of the digits of x = the sum of the digits of y & the sum of the digits of y = the sum of the digits of z. $\therefore$ The sum of the digits of x = the sum of the digits of z $\therefore x \sim z$ and $\sim$ is transitive

   $\therefore \sim$ is an equivalence relation

   (b) [1] How many distinct equivalence classes are there? Explain.

   One for each possible sum of digits

   $\therefore$ 18

3. Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function defined by $f(x) = 5x$.

(a) [2] Show that f is 1-1.

Suppose $f(x) = f(y)$

Then $\cancel{5}x = \cancel{5}y$

$\therefore$ $x = y$ and f is 1-1

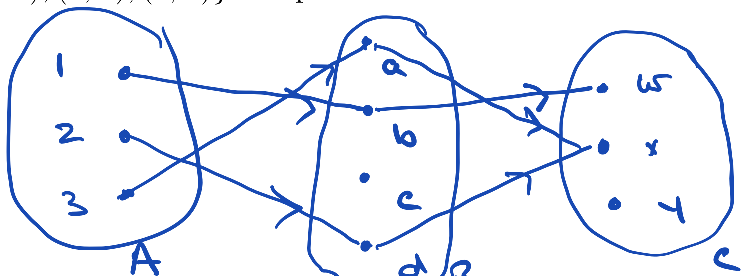
(b) [2] Show that f is not onto.

For any $x \in \mathbb{Z}$, $f(x)$ is a multiple of 5.

$\therefore$ There is no $x \in \mathbb{Z}$ s.t. $f(x) = 1$.

$\therefore$ f is not onto

4. [2] Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$ and $C = \{w, x, y\}$. Suppose $f : A \to B$ is $\{(1, b), (2, d), (3, a)\}$. How many possibilities are there for $g : B \to C$ if $g \circ f = \{(1, w), (2, x), (3, x)\}$? Explain.



Need: (b,w), (d,x), $(a,x) \in g$

3 choices for $g(c)$

$\therefore$ 3 possibilities

5. [2] Let $A = \{1, 2, 3, 4\}$ and $f : A \to A$ be $\{(1, 3), (2, 4), (3, 1), (4, 2)\}$. Answer each question **TRUE** (T) or **FALSE** (F). No reasons are necessary.

T f is 1-1.

T f is onto.

T f is invertible.

T $f \circ f = \iota_A$.