

202401 Math 122 A01 Quiz #5

#V _____

Name: _____

The time limit is 25 minutes. There are total of 15 marks available on the two pages. You may use a Sharp calculator with model number beginning EL510R. Except where indicated, you must show your work in order to receive full or partial credit.

1. [2] Let $\mathcal{R}$ be a relation on $A = \{1, 2, 3, 4\}$. Answer each question **TRUE** (T) or **FALSE** (F). No reasons are necessary.

___ If $1 \in A$ and $4 \in B$, then $1 \times 4 \in A \times B$.

___ If $|A| = m$ and $|B| = n$, then $|A \times B| = m^n$.

___ If $\mathcal{R}$ is symmetric and transitive, and $(3, 2) \in \mathcal{R}$, then $(2, 2) \in \mathcal{R}$.

___ If $\mathcal{R}$ is antisymmetric and transitive, and $(2, 4), (4, 3) \in \mathcal{R}$, then $(3, 2) \notin \mathcal{R}$.

2. Let $\sim$ be the relation on $A = \{10, 11, \dots, 99\}$ defined by $x \sim y \Leftrightarrow$ the sum of the digits in the decimal representation of x equals the sum of the digits in the decimal representation of y .

(a) [4] Prove that $\sim$ is an equivalence relation.

(b) [1] How many distinct equivalence classes are there? Explain.

3. Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be the function defined by $f(x) = 5x$.

(a) [2] Show that f is 1-1.

(b) [2] Show that f is not onto.

4. [2] Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$ and $C = \{w, x, y\}$. Suppose $f : A \rightarrow B$ is $\{(1, b), (2, d), (3, a)\}$. How many possibilities are there for $g : B \rightarrow C$ if $g \circ f = \{(1, w), (2, x), (3, x)\}$? Explain.

5. [2] Let $A = \{1, 2, 3, 4\}$ and $f : A \rightarrow A$ be $\{(1, 3), (2, 4), (3, 1), (4, 2)\}$. Answer each question **TRUE** (T) or **FALSE** (F). No reasons are necessary.

___ f is 1-1.

___ f is onto.

___ f is invertible.

___ $f \circ f = \iota_A$.