202201 Math 122 A03 Quiz #5

#V00: ____________ Name: Solutions

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   F The remainder when -42 is divided by -5 is -2.

   F For all $x \in \mathbb{R}$, $2\lfloor x \rfloor = \lfloor 2x \rfloor$. $2\lfloor 1/2 \rfloor = 0$, $\lfloor 2(1/2) \rfloor = 1$.

   T $5 \mid 0$. $0 = 0 \cdot 5$

   T $(\underbrace{1010}\underbrace{0101})_2 = (A5)_{16}$. $(1010)_2 = 8+2 = 10 = (A)_{16}$, $(0101)_2 = 4+1 = 5 = (5)_{16}$.

2. [3] Find the base 9 representation of 2022.

Apply the Division Algorithm! Then repeat with the quotients.

$2022 = 224 \cdot 9 + \underline{6}$ $\quad d_0$

$224 = 24 \cdot 9 + \underline{8}$ $\quad d_1$

$24 = 2 \cdot 9 + \underline{6}$ $\quad d_2$

$2 = 0 \cdot 9 + \underline{2}$ $\quad d_3$

Thus $2022 = \boxed{(2686)_9}$.

3. [3] Prove that if $a|b$ and $2b|5c$, then $a|10c$.

Since $a|b$, $b = ka$ for some $k \in \mathbb{Z}$.

Since $2b|5c$, $5c = 2bm$ for some $m \in \mathbb{Z}$.

Then $10c = 2(5c) = 2(2bm) = 4bm = 4(ka)m$
$= (4km)a$.

Since $4km \in \mathbb{Z}$, we have $a|10c$. ∎

4. [2] Let $n = 5^{55}7^{77}$. Explain why the Fundamental Theorem of Arithmetic implies that there does not exist an integer k such that $n = 14k$.

If $n=14k$, then $5^{55}7^{77} = 2\cdot 7k$ for some k.

The FTArith. says that $5^{55}7^{77}$ is the unique prime factorization of n, and so 2 is not a factor of n, contrary to $5^{55}7^{77} = 2\cdot 7k$. That's a contradiction, so n cannot equal $14k$ for some $k \in \mathbb{Z}$. ☒

5. [3] Use the Euclidean Algorithm to compute $d = \gcd(644, 154)$, and then use your work to find integers x and y such that $d = 644x + 154y$.

We have:

$$644 = 4\cdot 154 + 28$$
$$154 = 5\cdot 28 + 14$$
$$28 = 2\cdot 14 + \underset{\text{stop}}{0}$$

Thus $d = \gcd(644, 154) = \boxed{14.}$

We go backwards now:

$$14 = 154 - 5\cdot 28$$
$$= 154 - 5(644 - 4\cdot 154)$$
$$= \underline{(-5)}644 + \underline{(21)}154.$$

For $\boxed{x=-5,\ y=21,}$

$14 = 644x + 154y$.

6. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

only when $n/3 \in \mathbb{Z}$, i.e. $3|n$.

T For $n \in \mathbb{Z}$, if $\lfloor n/3 \rfloor = \lceil n/3 \rceil$ then n is a multiple of 3.

F For positive integers a and b, it is possible to simultaneously have $\gcd(a, b) > \sqrt{ab}$ and $\text{lcm}(a, b) > \sqrt{ab}$. $ab = \gcd\cdot\text{lcm}$

T If $a = 2^3 5^1 11^2$ and $b = 2^2 5^2 7^2$, then $\text{lcm}(a, b) = 2^3 5^2 7^2 11^2$. take max of exponents

F For any integers a and b, if $\gcd(a, b) = 5$ then neither a nor b are divisible by 25.

$a=5, b=25$: then $\gcd(a,b)=5$ but $25|b$.