201701 Math 122 [A01] Quiz #5

#V00:__________________ Name: Key

This quiz has 2 pages and 7 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary. Let $f : A \to B$ and $g : B \to C$ be functions.

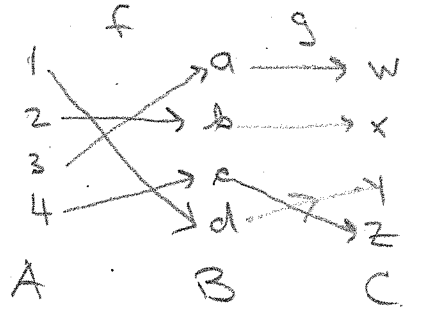
   F $g \circ f$ is a function from C to A.

   T If f and g are onto, so is $g \circ f$.

   T If f is not 1-1, then $g \circ f$ is not 1-1.

   T $\iota_C \circ g = g$

2. [2] Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, and $C = \{w, x, y, z\}$. Suppose $f : A \to B$ is $f = \{(2, b), (4, c), (1, d), (3, a)\}$. Find $g : B \to C$ such that $g \circ f = \{(1, y), (2, x), (3, w), (4, z)\}$.



$g = \{(d,y), (b,x), (a,w), (c,z)\}$

From the picture $g \circ f$ is as above.

3. Let $f : A \to B$ be a function. Suppose $g = \{(b, a) : (a, b) \in f\}$ is also a function.

   (a) [2] Explain why f must be 1-1 and onto. (Hint: what is needed for g to be a function?)

   If f is not 1-1, then there exist $a_1, a_2 \in A$ and $y \in B$ s.t. $(a_1, y), (a_2, y) \in f$. But then g has 2 ordered pairs that start with y $\Rightarrow\Leftarrow$.
   If f is not onto, then there exists $y \in B$ s.t. no ordered pair in f has 2nd component y. Then g has no ordered pair with 1st component y. $\Rightarrow\Leftarrow$

   (b) [1] Is $f^{-1} = g$? Why or why not?

   Yes, by definition $(b,a) \in f^{-1} \Leftrightarrow (a,b) \in f$. This is exactly the definition of g.

4. [2] Find the base b if $(121)_b = (224)_5$.

$(121)_b = b^2 + 2b + 1 = (b+1)^2$ $(224)_4 = 2 \cdot 25 + 2 \cdot 5 + 4 = 64$

Now, $(b+1)^2 = 64 \iff b+1 = \pm 8 \iff b = 7$ or -9

But -9 is not a base (in this class)

$\therefore b = 7$.

5. [2] Let a, b and c be integers such that $a + b = c$. Suppose d is an integer such that $d \mid a$ and $d \mid b$. Prove that $d \mid c$.

Since $d \mid a$, $a = kd$ for some $k \in \mathbb{Z}$.
Since $d \mid b$, $b = \ell d$ for some $\ell \in \mathbb{Z}$.
$\therefore c = a + b = kd + \ell d = (k + \ell)d$.
Since $k + \ell \in \mathbb{Z}$, $d \mid c$.

6. [2] Which theorem implies that there are no positive integers a and b so that $3^a = 5^b$? Why?

The Fundamental Theorem of Arithmetic.
If such positive integers exist, then $n = 3^a$ has two different prime factorizations 3^a and 5^b, a contradiction

7. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

F When -100 is divided by -7, the remainder is -5.
F $(1111000)_2 = (F0)_{16}$.
F The quantity 2^6 appears in the prime factorization of $n = 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$.
F $3^4 101^{100}$ is divisible by 3^5. (Note: 101 is prime.)