201709 Math 122 A01 Quiz #5

#V00:________________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except where explicitly noted, is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Let $X = \{a, \{b, c\}, d\}$. Use the blank to indicate whether each statement is true or false. No reasons are necessary.

   F $|X| = 4$.

   F $\{\emptyset\} \subseteq X$.

   T $\{b, c\} \in X$.

   T $\{d, a, \{b, c\}\} \subseteq X$.

2. [3] Let A, B be sets. Prove that $(A \cap B)^c \cap (A \setminus B)^c = A^c$. (Hint: there is a short proof using the Laws of Set Theory.)

$(A\cap B)^c \cap (A\setminus B)^c = (A\cap B)^c \cap (A\cap B^c)^c$ Known

$= (A^c \cup B^c) \cap (A^c \cup B^{\cancel{cc}})$ DeM. & Dbl comp

$= A^c \cup \cancel{(B^c \cap B)} \to \phi.$ Dist

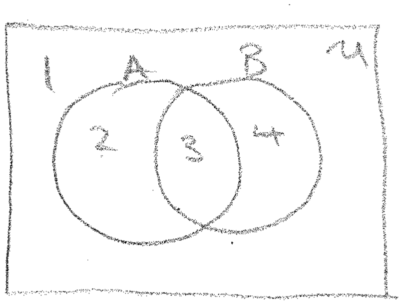
$= A^c \cup \phi$ Known

$= A^c$

3. [2] Use a Venn diagram to help obtain a counterexample to the statement:

$$A \oplus B^c = B \setminus A^c \quad \textit{for all sets } A \textit{ and } B.$$

Does your diagram suggest a relationship between the set on the LHS and the set on the RHS? Let $U = \{1,2,3,4\}$, $A = \{2,3\}$, $B = \{3,4\}$



Then $A \oplus B^c = \{2,3\} \oplus \{1,2\} = \{1,3\}$

$B \setminus A^c = \{3,4\} \setminus \{1,4\} = \{3\}$.

These sets are not equal, but the diagram & example suggest $B \setminus A^c \subseteq A \oplus B^c$

4. [3] Let A and B be sets such that $A \cup B = B$. Show that $A \subseteq B$ using an argument that begins with "*Take any* $x \in A$ ..."

Given $A \cup B = B$.
Take any $x \in A$.
Then $x \in A \cup B$ by def'n of union
Since $A \cup B = B$, $x \in B$
$\therefore A \subseteq B$

5. [3] Find the base 7 representation of 183.

$183 = 7 \times 26 + 1$
$26 = 7 \times 3 + 5$
$3 = 7 \times 0 + 3$

$\therefore 183 = (351)_7$

6. [2] Fill in each blank. No reasons are necessary.

(a) If $|A| = 6$, then A has $\underline{2^6 - 1 - 6}$ subsets of size at least 2.

(b) The number of subsets of $\{a, b, c, d, e\}$ that contain both a and b equals $\underline{2^3}$.

(c) If $|A| = 15$, $|B| = 20$ and $|A \cap B| = 8$, then $|A \cup B| = \underline{15 + 20 - 8}$.

(d) $(10|1110)_2 = \underline{(2\ E\ )}_{16}$.