201809 Math 122 A01 Quiz #5

#V00:________________ Name: Solutions

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

   T For $x \in \mathbb{R}$, $\lfloor x/2 \rfloor = \lceil x/2 \rceil$ if and only if x is an even integer.

   F When -102 is divided by -10, the remainder is -2.

   T The exponent of 2 in the prime factorization of $(8!) \cdot (9!)$ is 14.

   F $(1001110)_2 = (96)_{16}$.

2. [2] Find the base 9 representation of 2018.

$$2018 = 224 \cdot 9 + 2$$
$$224 = 24 \cdot 9 + 8$$
$$24 = 2 \cdot 9 + 6$$
$$2 = 0 \cdot 9 + 2$$

$\therefore 2018 = (2682)_9$

3. [3] Let $a, b, d \in \mathbb{Z}$. Prove that if $d \mid a$ and $d \mid b$, then $d^2 \mid ab$.

Suppose $d \mid a$ and $d \mid b$.
Then there exist integers k_1 & k_2 such that

$$a = dk_1 \quad \& \quad b = dk_2$$

$\therefore ab = dk_1 dk_2 = d^2(k_1k_2)$.
Since $k_1k_2 \in \mathbb{Z}$, $d^2 \mid ab$.

4. [2] Let $n = 2^{100}3^{200}$. Explain why the Fundamental Theorem of Arithmetic implies that there is no integer k such that $15k = n$.

The prime factorization of 15k has a 5.
The " " " n has no 5.
By the FTA, n has a unique prime factorization.
$\therefore$ $15k \neq n$ for any integer k.

5. (a) [3] Use the Euclidean Algorithm to find $d = gcd(824, 122)$, and then use your work to find integers x and y such that $122x + 824y = d$.

$$824 = 6 \times 122 + 92$$
$$122 = 1 \times 92 + 30$$
$$92 = 3 \times 30 + 2 \leftarrow d = gcd(a,b)$$
$$30 = 15 \times 2 + 0$$

$$\begin{aligned} 2 &= 92 - 3\times 30 \\ &= 92 - 3(122 - 92) = 4\times 92 - 3\times 122 \\ &= 4\times(824 - 6\times 122) - 3\times 122 \\ &= 824\times 4 - 27\times 122 \\ &= 824 \times \underbrace{4}_{x} + 122\underbrace{(-27)}_{y} \end{aligned}$$

(b) [1] Use your answer from (a) to find $lcm(824, 122)$.

$$lcm(824, 122) = \frac{824 \times 122}{2\ \cancel{gcd(824,122)}} = 50264$$

6. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

F There is an integer b such that $(23)_b = (32)_4$.

T If $a, b \in \mathbb{Z}$ and $gcd(a, b) = 4$, then there are integers x, y such that $ax + by = 12$.

T $0 \mid 0$.

T If $a = 2^4 \cdot 5^3 \cdot 7$ and $b = 2^2 \cdot 5^6 \cdot 11$, then $gcd(a, b) = 2^2 \cdot 5^3$.