201909 Math 122 A01 Quiz #5

#V00:________________ Name: Solutions

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, and may even be useful. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

   T For all $n \in \mathbb{Z}$, $2\lfloor n/2 \rfloor = n$ if and only if n is even.

   F $(1011001)_2 = (B1)_{16}$.

   T The exponent of 3 in the prime factorization of 9! is 4.

   F $0 \mid 3$.

2. [2] Find the base 11 representation of 1219. Use δ for the base 11 digit corresponding to ten.

$$1219 = 110 \times 11 + 9$$
$$110 = 10 \times 11 + 0$$
$$10 = 0 \times 11 + \delta$$

$\therefore$ $1219 = (\delta 0 9)_{11}$

3. [3] Let $a, d \in \mathbb{Z}$. Prove that if $d \mid a$, then $3d \mid 6a$.

Suppose $d \mid a$.

$\therefore$ There exists an integer k s.t. $dk = a$.

$\therefore$ $6a = 6(dk) = (3d)(2k)$.

Since $2k$ is an integer, $3d \mid 6a$

4. (a) [2] Suppose there are integers a and b such that $3a = 2b$. Use the Fundamental Theorem of Arithmetic to explain why b must be a multiple of 3.

By the FTA the numbers 3a and 2b must have the same prime factorization
$\therefore$ 3 is a prime factor of b
$\therefore$ b is a multiple of 3

(b) [1] Explain why there are no positive integers a and b such that $5^a = 7^b$.

By the FTA the number 5^a, $a>0$, has only one prime factorization. (It can not also be factored as a product of 7s.)

5. (a) [2] Use the Euclidean Algorithm to find $d = gcd(1254, 228)$.

$1254 = 5 \times 228 + 114 \leftarrow$
$228 = 2 \times 114 + 0$

$\therefore gcd(1254, 228) = 114$

(b) [1] Use your answer from (a) to find $lcm(1254, 228)$.

$$lcm(1254, 228) = \frac{1254 \times 228}{114} = 2508$$

6. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

T If $a, b \in \mathbb{Z}$ are relatively prime, then there exist $x, y \in \mathbb{Z}$ such that $ax + by = 2$.

T $100 \equiv -8 \pmod{12}$.

T $23 \cdot 122 + 3 \cdot 10^9 \equiv 9 \pmod{11}$.

F If $a \in \mathbb{Z}$ and $a \equiv 5 \pmod 9$, then the remainder when a is divided by 9 equals 4.