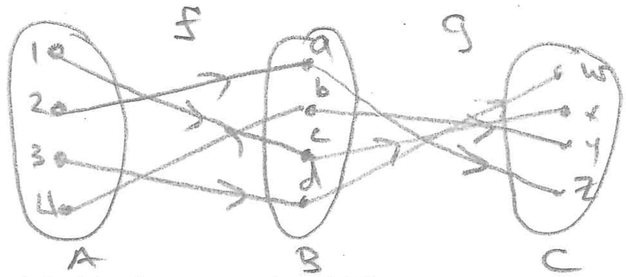
201609 Math 122 [A03] Quiz #5

#V00:________________ Name: Key

This quiz has 2 pages and 7 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but calculators will not help with these questions! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, and $C = \{w, x, y, z\}$. Suppose that $f : A \to B$ is $f = \{(2, a), (4, b), (1, c), (3, d)\}$, and $g : B \to C$ is $g = \{(a, z), (b, y), (c, x), (d, w)\}$. If $g \circ f$ is invertible, then find its inverse. If $g \circ f$ is not invertible, then explain why not.



$g \circ f$ is 1-1 & onto
$\therefore$ invertible
$(g \circ f)^{-1} = \{(w,3),(x,1),(y,4),(z,2)\}$.

2. [2] Find the base 7 representation of 433.

$433 = 61 \times 7 + 6$
$61 = 8 \times 7 + 5$
$8 = 1 \times 7 + 1$
$1 = 0 \times 7 + 1$

$\therefore 433 = (1156)_7$

3. [2] Let a, b and c be integers such that $a + b = c$. Suppose d is an integer such that $d \mid a$ and $d \mid b$. Prove that $d \mid c$.

Since $d \mid a$, $a = kd$ for some $k \in \mathbb{Z}$.
Since $d \mid b$, $b = ld$ " " $l \in \mathbb{Z}$.
$\therefore c = a + b = kd + ld = (k+l)d$.
Since $k + l \in \mathbb{Z}$, $d \mid c$.

4. [2] Find the integers a, b and c if $2^a 5^{10} = 2^7 3^b 5^c$. Which theorem are you using in your answer?

By the Fundamental Theorem of Arithmetic,

$a = 7$, $b = 0$, $c = 10$

5. [2] Find the prime factorization of $a = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, and of $b = (12 \times 50)^3$, and then use these to find $gcd(a, b)$ and $lcm(a, b)$.

$a = 2^7 3^2 5^1 7^1$
$b = 2^9 \quad 5^6$

$\therefore\ gcd(a,b) = 2^7 5^1$
$lcm(a,b) = 2^9 3^2 5^6 7^1$

6. Let $n \geq 2$ be a positive integer.

(a) [1] Suppose that $p \leq n$ is a prime number. What is the remainder when $(1 \times 2 \times \cdots \times p \times \cdots n) + 1$ is divided by p?

One, as $p \mid 1 \times 2 \times \cdots \times n$

(b) [1] Explain why part (a) implies that $(1 \times 2 \times \cdots \times n) + 1$ has a prime divisor greater than n.

Every prime less than n leaves a remainder of 1 when divided into $1 \times 2 \times \cdots \times n + 1$ (by (a))

(c) [1] Explain why part (b) implies that there are infinitely many prime numbers.

It says that for any n there is a prime larger than n.
$\therefore$ For any prime, there is a larger prime.
(# primes finite $\Rightarrow \exists$ largest prime.)

7. [2] Use the blank to indicate whether each statement is true or false. No reasons are necessary.

T If $b > 2$ then $(121)_b$ is the square of an integer.
T If $k \equiv 3 \pmod 5$, then when $3k - 1$ is divided by 5, the remainder is 4.
T $(2503)_9 \equiv 3 \pmod 9$.
T If $x \equiv 3 \pmod 8$ and $y \equiv 5 \pmod 8$ then $xy \equiv 7 \pmod 8$.