201801 Math 122 A01 Quiz #5

#V00:________________ Name: Key

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

   F If $x \in \mathbb{R}$ then $\lfloor x/3 \rfloor < \lceil x/3 \rceil$. eg $x = 3$

   F When -102 is divided by -10, the remainder is 2. $-102 = -10 \times 11 + 8$

   F $(7890)_9$ is the base 9 representation of some integer. 9 isn't a base 9 digit

   T $(30\,32)_4 = (1100\,1110)_2$.

2. [2] Find the base 16 representation of (the base 10 number) 2018. Remember that the letters $A, B, \ldots, F$ are used to represent the base 16 digits $10, 11, \ldots, 16$, respectively.

$$2018 = 126 \times 16 + 2$$
$$126 = 7 \times 16 + 14 \to E$$
$$7 = 0 \times 16 + 7$$
$$0 = 0 \times 16 + 0$$

$\therefore 2018 = (7E2)_{16}$.

3. [3] Let $a, b, d \in \mathbb{Z}$. Prove that if $d \mid a$ and $d \mid b$, then $d \mid 3a - 4b$.

Given: $d \mid a$ and $d \mid b$

$\therefore \exists k \in \mathbb{Z}$ s.t. $dk = a$

& $\exists \ell \in \mathbb{Z}$ s.t. $d\ell = b$

$\therefore 3a - 4b = 3dk - 4d\ell$
$= d(3k - 4\ell)$

Since $k, \ell \in \mathbb{Z}$, $3k - 4\ell \in \mathbb{Z}$.

$\therefore d \mid 3a - 4b$.

4. [2] Let $a \in \mathbb{Z}$. Suppose that p is a prime number such that $p \mid a$. Explain why the Fundamental Theorem of Arithmetic implies that $p^2 \mid a^2$.

If $p|a$ then, by the FTA, p is a prime factor of a. When a is squared, every exponent in its prime factorization is doubled
$\therefore$ p has exponent at least 2 in the prime factorization of a^2
$\therefore$ $p^2 | a^2$

5. (a) [3] Use the Euclidean Algorithm to find $d = gcd(122, 580)$, and then use your work to find integers x and y such that $122x + 580y = d$.

$580 = 122 \times 4 + 92$
$122 = 92 \times 1 + 30$
$92 = 30 \times 3 + 2 \leftarrow \therefore gcd(122,580) = 2$
$30 = 15 \times 2 + 0$

$2 = 92 - 30 \times 3$
$= 92 - 3 \times (122 - 92) = 4 \times 92 - 3 \times 122$
$= 4 \times (580 - 122 \times 4) - 3 \times 122$
$= 4 \times 580 - 19 \times 122$
$= \underbrace{580(4)}_{x} + \underbrace{122(-19)}_{y}$

(b) [1] Find $lcm(122, 580)$.

$L = \dfrac{122 \times 580}{gcd(122,580)} = \dfrac{70760}{2} = 35\,380$

6. [2] Answer each question **True (T)** or **False (F)**. No justification is needed.

F The exponent of 3 in the prime factorization of 12! equals 4. $\times 10$ $2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12$

F If $a, b \in \mathbb{Z}$ and there are integers x, y such that $ax + by = 3$, then $gcd(a, b) = 3$. it divides 3

F There are integers a, b such that $3 \cdot 2^a = 2 \cdot 5^b$. by UFT

F If $a = 2^4 \cdot 5^3 \cdot 7$ and $b = 2^2 \cdot 5^6 \cdot 11$, then $lcm(a, b) = 2^2 \cdot 5^3 \cdot 7 \cdot 11$.

$2^4 5^6 7^1 11$