201809 Math 122 Quiz #6

This is a take-home quiz. It is due no later than 2:30 PM on Wednesday, December 5. Late quizzes will not be accepted except in documented cases of illness, emergency, accident, or affliction.

There are 4 ways your solutions can be submitted: (1) to Gary in DTB A442 between 11 AM and 12 noon; (2) to Michelle in DTB A104 between 11:30 and 12:30; (3) to Michelle in DTB A437 between 1 PM and 2 PM; or (4) to Michelle at the start of her 2:30 class in COR A120. **Please clearly indicate your section number on your answer paper.**

This quiz is to be done individually. You may consult any pre-existing resources on the course page or elsewhere. Any form of collaboration or communication between persons is not permitted.

There are 5 questions with marks as shown, and a total of 15 marks available. For each question, it is necessary to show clearly organized work in order to receive full or partial credit. Answers must be written in your own words in a way that reflects your own understanding.

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1. [4] Answer each question, and provide details to support your answer.
   (a) If $k \equiv -15 \pmod{11}$ what is the remainder when $5k^2 - 122k$ is divided by 11?
   (b) Is it true that if $x^2 \equiv y^2 \pmod 8$, then $x \equiv y \pmod 4$?
2. [4] Answer each question, and provide details to support your answer.
   (a) Is the function $f : \mathbb{Q} \to \mathbb{Q}$ defined by $f(x) = 4x - 7$ one-to-one?
   (b) Is the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 4x + 12$ onto?
3. [3] Let $\mathcal{R}$ be the relation on $A = \{10, 11, \ldots, 99\}$ defined by $x\mathcal{R}y \Leftrightarrow$ the sum of the digits in the decimal representation of x equals the sum of the digits in the decimal representation of y. For example, $(44, 80) \in \mathcal{R}$ because $4 + 4 = 8 + 0$.
   (a) Prove that $\mathcal{R}$ is an equivalence relation.
   (b) How many distinct equivalence classes are there? Explain.
4. [2] Suppose $f : \{1, 2, 3\} \to \{a, b, c, d\}$ is $f = \{(1, b), (2, d), (3, a)\}$. Find a function $g : \{a, b, c, d\} \to \{4, 5, 6\}$ such that $g \circ f$ is onto. How many different functions g have the property that $g \circ f$ is onto?
5. [2] Answer each question, and provide details to support your answer.
   (a) Let $\mathcal{R}$ be a relation on $\{1, 2, 3\}$ which is reflexive, antisymmetric, and transitive. Suppose $(2, 1), (1, 3) \in \mathcal{R}$. Write $\mathcal{R}$ as a set of ordered pairs. If there are ordered pairs not in your set, justify why they are not in $\mathcal{R}$.
   (b) Suppose $\mathcal{R}$ is reflexive, symmetric and antisymmetric relation on $A = \{1, 2, 3\}$. Write $\mathcal{R}$ as a set of ordered pairs. If there are ordered pairs not in your set, justify why they are not in $\mathcal{R}$.