202201 Math 122 A03 Quiz #6

#V00: _____ Name: _____

This quiz has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   ___ For non-zero integers a and b, if there are integers x, y such that $ax + by = 2$, then a and b are always relatively prime.

   ___ If $k \equiv 7 \bmod 8$, then $-2k + 14$ is divisible by 8.

   ___ For any integer $b > 1$ and integers d_0 and d_1, $d_1 b + d_0$ is divisible by $b - 1$ if and only if $d_1 + d_0$ is divisible by $b - 1$.

   ___ An anti-symmetric relation $\mathcal{R}$ on $\{1, 2, 3\}$ can contain both $(1, 3)$ and $(3, 1)$.

2. [3] Prove that if $x \equiv 4 \bmod 14$, then $4x + 1 \equiv 3 \bmod 7$.

3. [3] Find the ones digit of 23^{37} in base 9 without directly computing 23^{37}.

4. [3] Let $A = \{1, 2, 3, 4, 5\}$. The set $\mathcal{R} = \{(a, b) \in A \times A \ : \ b - a \in \{0, 2\}\}$ is a relation on A. Write out $\mathcal{R}$ explicitly; then, determine how to add *as few ordered pairs as possible* to $\mathcal{R}$ to get an equivalence relation $\mathcal{E}$ on A (you don't need to write $\mathcal{E}$ out after that, just the pairs to add). How many equivalence classes does $\mathcal{E}$ have?

5. [2] Let $B = \{a, b, c, d\}$. The set $f = \big\{(a, c), (b, b), (c, c), (d, a)\big\}$ is a function from B to B. Find counter-examples (with justification) to show that f is...

   - not one-to-one:

   - not onto:

6. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   ___ For $A = \{1, 2\}$ and $B = \{1, 3\}$, $(A \times B) \cap (B \times A) \neq \emptyset$.

   ___ Congruence modulo 4 partitions $\mathbb{Z}$ into four equivalence classes.

   ___ Let A be a non-empty set. Then $A \times A$ is an equivalence relation on A.

   ___ If $f : A \to B$ is one-to-one, then $A = \text{range}(f)$.